Altruism and Risk Sharing in Networks

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Abstract: We provide the first analysis of the risk sharing implications of altruism networks. Agents are embedded in a fixed network and care about each other. We explore whether altruistic transfers help smooth consumption and how this depends on the shape of the network. We find that altruism networks have a first-order impact on risk. Altruistic transfers generate efficient insurance when the network of perfect altruistic ties is strongly connected. We uncover two specific empirical implications of altruism networks. First, bridges can generate good overall risk sharing and, more generally, the quality of informal insurance depends on the average path length of the network. Second, large shocks are well-insured by connected altruism networks. By contrast, large shocks tend to be badly insured in models of informal insurance with frictions. We characterize what happens for shocks that leave the structure of giving relationships unchanged. We further explore the relationship between consumption variance and centrality, correlation in consumption streams across agents and the impact of adding links.

Keywords: Altruism, Networks, Risk Sharing, Informal Insurance.

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1 Introduction

Informal safety nets are key to helping people cope with negative shocks, especially in areas with little or no access to formal insurance.[1] Applied economists have extensively studied the effectiveness of informal risk sharing arrangements. Townsend (1994) tested and rejected the hypothesis of efficient risk sharing in villages of rural India. He found that, even though risk sharing inefficiency is surprisingly low, household income remains a significant determinant of consumption. These findings, confirmed by a large literature, have led researchers to model informal transfers as insurance contracts subject to frictions. Ligon, Thomas and Worrall (2002) characterize constrained efficient risk sharing contracts in a dynamic framework with limited commitment.[2] Different frictions, such as limited commitment, moral hazard or hidden income, have different dynamic implications which can be tested on panel consumption data, e.g., Kinnan (2019).

The literature on informal insurance contracts has, however, neglected two key features of informal transfers. First, there is expanding empirical evidence that informal transfers flow through family and social networks.[3] Exogenous or endogenous networks thus appear to play a central role in the organization of risk sharing. A few recent papers study the interplay between network structure and contracting frictions.[4] Ambrus, Mobius & Szeidl (2014) characterize constrained efficient risk sharing contracts in a static setup with limited commitment where social ties can be used as social collateral and destroyed to punish deviations. The ties' values thus impose capacity constraints on transfers. However, no attempt has yet been made to fully integrate networks into the dynamic analysis of insurance contracts with frictions.

Second, the literature on informal insurance contracts focuses on one motive behind transfers: self-interested gains from trade. Yet, there are other motives that deserve at-

[1] See, for instance, Chapter 6 in Banerjee and Duflo (2011).

[2] See also, among others, Ábrahám & Laczó (2017), Coate & Ravallion (1993), Dubois, Jullien & Magnac (2008), Kocherlakota (1996), Laczó (2015), Ligon (1998), Morten (2019).

[3] Applied researchers collect and analyze detailed data on actual transfers and helping relationships between individuals and households, see Fafchamps & Lund (2003), De Weerdt & Dercon (2006), Fafchamps & Gubert (2007), Banerjee et al. (2013), Jack & Suri (2014).

[4] We review this emerging literature below.

tention. Informal transfers are, to a large extent, motivated by altruism. Individuals give support to others they care about and, in particular, to their family and friends in need.[5] Altruistic transfers flow from households with positive income shocks to relatives with negative income shocks, unhindered by contracting frictions. Transfers motivated by altruism can thus help smooth consumption, even in the absence of insurance contracts. Our goal in this paper is to study this benchmark. In Section 6, we discuss how future research could integrate altruism, social networks and insurance contracts with frictions.

We provide the first analysis of the risk sharing implications of altruism networks. We introduce stochastic incomes into the model of altruism in networks analyzed in Bourlès, Bramoullé & Perez-Richet (2017). Agents care about each other and the altruism network describes the structure of social preferences. For each realization of incomes, agents play a Nash equilibrium of the game of transfers. They do not have access to formal or informal insurance contracts, for instance due to large contracting frictions. Our objective is to understand how altruistic transfers affect the risk faced by the agents. Do altruism networks help smooth consumption? How does the structure of the network affect this? Do the empirical implications of altruism in networks differ from those of other models of informal transfers?

We find that altruism networks have a first-order impact on risk and yield distinct implications. In line with Becker (1974)'s intuition, altruistic transfers often mimic insurance contracts.[6] Altruistic agents tend to give to others when rich and receive from others when poor, reducing consumption variability. In our first main result, we show that altruistic transfers can even yield efficient risk sharing. This happens for any utility functions and any income distribution if and only if the network of perfect altruistic ties is strongly connected.[7] Some altruistic relationships must be very strong, but the overall network can

[5] Direct and indirect evidence that some informal transfers are altruistically motivated can be found in Foster & Rosenzweig (2001), Leider et al. (2009), De Weerdt & Fafchamps (2011), Karner (2012), Ligon & Schechter (2012). Altruism likely explains a large proportion of remittances, a main source of income for many poor households, see Yang (2011). Remittances generally increase following a negative shock and hence help smooth consumption, e.g., Yang & Choi (2007), Jack & Suri (2014).

[6] In a context of household decision-making, "The head's concern about the welfare of other members provides each, including the head, with some insurance against disasters.", Becker (1974, p.1076).

[7] An altruistic relationship between agent i and agent j is perfect when agent i cares as much about her own well-being as about agent j's well-being. The network of perfect altruistic ties is strongly connected

be sparse. In this case, consumption patterns resulting from altruistic transfers cannot be distinguished from those induced by frictionless insurance contracts.

Identification becomes possible in the domain of inefficient risk sharing. We uncover two specific empirical implications of altruism networks.[8] We find, first, that bridges play a critical role in overall risk sharing. One strong tie between two separate communities generates large spillovers and can lead to good overall risk sharing. More generally, the quality of informal insurance induced by altruistic transfers depends on the average path length of the network. These structural predictions are very different from those obtained by Ambrus, Mobius & Szeidl (2014) in the social collateral framework. When links have capacity constraints, a bridge generally has little impact and the quality of informal insurance depends on the expansiveness of the network rather than on its average path length. For example, consider rural Indian villages where economic and social interactions are structured along caste lines. The model of altruism in networks predicts that a few inter-caste marriages can drastically change the overall patterns of informal transfers and consumption.

Second, we find that a large negative shock on one agent is well-insured when the altruism network is connected. The whole community is involved, directly or indirectly, in supporting the agent. By contrast, we show that a large negative shock on one agent is badly insured in the social collateral framework. Large transfer requirements saturate capacity constraints. Once transfers reach their upper bound, further increases in shock size are fully borne by the agent. In general, large shocks are not well-insured by informal insurance contracts subject to frictions. Incentive compatibility typically imposes upper bounds on transfers, limiting insurance against large shocks.[9]

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if any two agents are indirectly connected via a path of perfect altruistic relationships, see Section 3.

[8] We obtain these results under the assumption that agents have a common utility function which displays Constant Absolute Risk Aversion (CARA). We discuss extensions to other classes of utility functions in Section 3.

[9] Another differential implication is that, in some situations, transfers always flow in the same direction under altruism. This cannot happen with informal insurance contracts, which rely on transfer reversals. In an unpublished PhD dissertation, Karner (2012) derives differing implications of altruism and informal insurance contracts and tests these implications on data from Indonesia. He finds that transfers tend to persistently flow in the same directions, consistently with altruism. We thank Dilip Mookherjee for bringing this interesting work to our attention.

We obtain a number of further results that provide insights into the risk sharing implications of altruistic transfers. We first characterize what happens for income shocks that leave the equilibrium structure of giving relationships unchanged. This generically covers any small shocks, as well as some large shocks. In these cases, we show that altruistic transfers yield constrained efficient risk sharing within the weak components of the transfer network.[10] Conversely, constrained efficiency generically holds only when giving relationships are invariant across income realizations. We then analyze consumption correlation. We show that if incomes are independent, altruistic transfers generate positive correlation in consumption streams across agents. Finally, we investigate the impact of the network structure through numerical simulations. With iid incomes, we find that a more central agent tends to have less variable consumption and that consumption correlation between two agents tends to decrease with network distance. We also find that adding an altruistic tie to the network can decrease or increase the consumption variance of indirect neighbors.

Our analysis contributes to a large literature on informal insurance. Applied researchers have tested the assumption of efficient risk sharing in many different contexts, see e.g. Townsend (1994), Mazzoco & Saini (2012). A common finding is that risk sharing is inefficient, but not too inefficient. Researchers have explained this finding, and further explored the properties of risk sharing arrangements, with models of informal insurance contracts subject to frictions. This literature, however, generally ignores altruistic motives and the role played by social networks as channels of informal transfers. We consider a different benchmark here. We provide the first analysis of the risk sharing implications of altruism networks, when ex-ante contracting is very frictional or altogether impossible. We notably show that altruism networks can generate good but imperfect risk sharing and identify specific empirical implications of altruism in networks.

Our analysis also contributes to a recent theoretical literature on informal insurance in networks.[11] Ambrus, Mobius & Szeidl (2014) characterize Pareto-constrained risk sharing

[10] Two agents are in the same weak component of the transfer network if they are indirectly connected by a path of giving or receiving transfers, see Section 4.

[11] One branch of this literature looks at network formation and network stability, e.g., Bloch, Genicot & Ray (2007), Bramoullé & Kranton (2007a, 2007b).

contracts when links can be used as social collateral and link values constrain transfer flows. They find that more expansive networks provide better insurance. In a related framework, Ambrus, Milan & Gao (2019) characterize Pareto-constrained risk sharing arrangements under local informational constraints. Jackson, Rodriguez-Barraquer & Tan (2012) analyze the conditions under which networks can sustain the exchange of discrete favors over time. They find that links must be "supported" by a common friend in equilibrium networks. We introduce stochastic incomes to the setup of Bourlès, Bramoullé & Perez-Richet (2017). Agents are embedded in a fixed network of altruism and, once incomes are realized, play a Nash equilibrium of the transfer game. We identify key structural properties of risk sharing induced by altruistic transfers. In particular, informal insurance tends to be better when the altruism network has lower average path length, a very different property from expansiveness and support.[12] Bridges notably play a critical role under altruism, but not in the other models.

Finally, our analysis advances the economics of altruism pioneered by Becker (1974) and Barro (1974). Economic studies of altruism consider either small groups of completely connected agents (e.g. Alger & Weibull (2010), Bernheim & Stark (1988), Bruce & Waldman (1991)) or linear dynasties (e.g. Altig & Davis (1992), Galperti & Strulovici (2017), Laitner (1988)). However, these structures are not realistic. As is well-known from human genealogy and argued early on by Bernheim & Bagwell (1988), family ties form complex networks. Bourlès, Bramoullé & Perez-Richet (2017) introduce networks into a model of altruism à la Becker, with non-stochastic incomes. We build on this previous analysis and study whether and how altruism networks affect risk.

The remainder of the paper is organized as follows. We introduce the model of altruism in networks with stochastic incomes in Section 2. We analyze arbitrary shocks in Section 3. We characterize what happens for income shocks leaving the structure of giving relationships unchanged in Section 4. We investigate structural effects in Section 5 and conclude

[12] Expansiveness measures the number of connections that groups of agents have with the rest of the network, relative to group size. A link connecting two agents is supported if these two agents have a common friend. Average path length measures the average network distance between any two agents. These three measures capture distinct features of a network's structure.

in Section 6.

2 Setup

We introduce stochastic incomes into the model of altruism in networks analyzed in Bourlès, Bramoullé & Perez-Richet (2017). Society is composed of $n \geq 2$ agents who may care about each other. Incomes are stochastic. Once incomes are realized, informal transfers are obtained as Nash equilibria of a non-cooperative game of transfers. We first describe how transfers are determined conditional on realized incomes. We then introduce risk and the classical notion of efficient insurance.

2.1 Transfers conditional on incomes

Agent i has income $y_i^0 \geq 0$ and can give $t_{ij} \geq 0$ to agent j. By convention, $t_{ii} = 0$. The collection of bilateral transfers $\mathbf{T} \in \mathbb{R}_+^{n^2}$ defines a network of transfers. Income after transfers, or consumption, y_i is equal to

$$y_i = y_i^0 - \sum_j t_{ij} + \sum_k t_{ki} \tag{1}$$

where $\sum_j t_{ij}$ represents overall transfers made by i and $\sum_k t_{ki}$ overall transfers received by i. Private transfers redistribute income among agents and aggregate income is conserved: $\sum_i y_i = \sum_i y_i^0$.

Agent i chooses her transfers to maximize her altruistic utility:

$$v_i(\boldsymbol{y}) = u_i(y_i) + \sum_{j \neq i} \alpha_{ij} u_j(y_j) \tag{2}$$

under the following assumptions. Private utility $u_i : \mathbb{R} \to \mathbb{R}$ is twice differentiable and satisfies $u_i' > 0$, $u_i'' < 0$ and $\lim_{y \to \infty} u_i'(y) = 0$. Coefficient $\alpha_{ij} \in [0, 1]$ captures how much i cares about j's private well-being. By convention $\alpha_{ii} = 1$. The *altruism network*

$\boldsymbol{\alpha} = (\alpha_{ij})_{i,j=1}^n$ represents the structure of social preferences.[13] In addition, we assume that

$$\forall i, j, \forall y, u_i'(y) \geq \alpha_{ij} u_j'(y) \tag{3}$$

which guarantees that an agent's transfer to a friend never makes this friend richer than her. When agents have the same utility functions, this assumption simply reduces to $\alpha_{ij} \leq 1$.

In a Nash equilibrium, each agent chooses her transfers to maximize her altruistic utility conditional on transfers made by others.[14] Transfer network $\mathbf{T} \in \mathbb{R}_+^{n^2}$ is a Nash equilibrium if and only if the following conditions are satisfied:

$$\forall i, j, u_i'(y_i) \geq \alpha_{ij} u_j'(y_j) \text{ and } t_{ij} > 0 \Rightarrow u_i'(y_i) = \alpha_{ij} u_j'(y_j) \tag{4}$$

In particular under common CARA utilities $u_i(y) = -e^{-Ay}$, equilibrium conditions become: $\forall i, j, y_i \leq y_j - \ln(\alpha_{ij})/A$ and $t_{ij} > 0 \Rightarrow y_i = y_j - \ln(\alpha_{ij})/A$.

Our analysis builds on equilibrium properties established in our previous paper.[15] In particular, an equilibrium always exists, equilibrium consumption is unique, and the network of equilibrium transfers is generically unique and has a forest structure. Formally, $\mathbf{T}$ has a forest structure when it contains no non-directed cycle, i.e., sets of agents $i_1, i_2, ..., i_l = i_1$ such that $\forall s < l$, $t_{i_s i_{s+1}} > 0$ or $t_{i_{s+1} i_s} > 0$.

Proposition 1 *(Bourlès, Bramoullé & Perez-Richet 2017) A Nash equilibrium exists. Equilibrium consumption* $\boldsymbol{y}$ *is unique and continuous in* $\boldsymbol{y}^0$ *and* $\boldsymbol{\alpha}$. *Generically in* $\boldsymbol{\alpha}$, *the network of equilibrium transfers is unique and is a forest.*

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[13] These preferences could be exogenously given, or could be generated by primitive preferences where agents care about others' private and social utilities, see Bourlès, Bramoullé & Perez-Richet (2017, p.678).

[14] Note that to compute her best response, an agent only needs to know the levels of income after transfers of the other agents she cares about. She does not need to know the precise transfers made by or to others, nor what happens in distant parts of the network.

[15] Our assumptions differ slightly from the assumptions made in Bourlès, Bramoullé & Perez-Richet (2017), to cover situations where altruism may be perfect and $\alpha_{ij} = 1$. We describe in Appendix how our previous results generalize to this extended setup.

2.2 Stochastic incomes

We now consider stochastic incomes. Following each income realization, agents make equilibrium transfers to each other. Proposition 1 ensures that there is a well-defined mapping from incomes to consumption. Let $\tilde{\boldsymbol{y}}^0$ denote the stochastic income profile and $\tilde{\boldsymbol{y}}$ the resulting stochastic consumption profile.[16]

To illustrate how altruistic transfers affect risk, consider the following simple example. Two agents care about each other with $\alpha_{12} = \alpha_{21} = \alpha$. They have common CARA utilities $u(y) = -e^{-y}$. Let $c = -\ln(\alpha)$. Agents' incomes are iid with binary distribution: $y_i^0 = \mu - \sigma$ with probability 1/2 and $y_i^0 = \mu + \sigma$ with probability 1/2, with $\sigma > c/2$. When one agent has a positive shock and the other a negative shock, the lucky agent makes a positive transfer to the unlucky one. Altruistic transfers lead to the following stochastic consumption: $(y_1, y_2) = (\mu - c/2, \mu + c/2)$ with probability 1/4, $(\mu + c/2, \mu - c/2)$ with probability 1/4, $(\mu - \sigma, \mu - \sigma)$ with probability 1/4, $(\mu + \sigma, \mu + \sigma)$ with probability 1/4.

In this example, consumption $\tilde{\boldsymbol{y}}$ is less risky than income $\tilde{\boldsymbol{y}}^0$ for Second-Order Stochastic Dominance. The reason is that altruism entails giving money when rich and receiving money when poor. Altruistic transfers in this case mimic a classical insurance scheme. While informal insurance provided by altruistic transfers is generally imperfect, $\tilde{\boldsymbol{y}}$ becomes less and less risky as α increases and idiosyncratic risks are fully eliminated when $\alpha = 1$. In the rest of the paper, we study how these effects and intuitions extend to complex networks and risks.

Our analysis relies on the classical notion of efficient insurance, see e.g. Gollier (2001).

Definition 1 *Informal transfers generate efficient insurance if there exist Pareto weights* $\boldsymbol{\lambda} \geq \mathbf{0}$, $\boldsymbol{\lambda} \neq \mathbf{0}$ *such that consumption* $\tilde{\boldsymbol{y}}$ *solves*

$$\max_{\substack{\boldsymbol{y} \\ \sum_i y_i = \sum_i y_i^0}} \sum_i \lambda_i \mathbb{E} u_i(y_i)$$

Efficient insurance is a central notion, describing the ex-ante Pareto frontier with respect

[16] Throughout the paper, we denote random variables with tilde and specific realizations of these random variables without tilde.

to private utilities. It provides the conceptual foundation of a large empirical literature, following Townsend (1994), which attempts to assess the extent of actual insurance in real contexts. Note that $\sum_i \mu_i \mathbb{E}v_i = \sum_i(\sum_j \alpha_{ji}\mu_j)\mathbb{E}u_i$. Therefore, a Pareto optimum with respect to expected altruistic utilities always generates efficient insurance. The converse may not be true, however, and efficient insurance situations may not constitute altruistic Pareto optima.[17]

Let us next recall some well-known properties of efficient insurance. When $\boldsymbol{\lambda} > \mathbf{0}$, efficient insurance is such that $u_i'(y_i)/u_j'(y_j) = \lambda_j/\lambda_i$ for every income realization $\boldsymbol{y}^0$. The ratio of two agents' marginal utilities is constant across states of the world. Define $\bar{y}^0 = (\sum_i y_i^0)/n$. When agents have common utilities and equal Pareto weights $\lambda_i = \lambda_j = \lambda$, this leads to equal income sharing $y_i = \bar{y}^0$. When agents have CARA utilities and under normalization $\sum_k \ln(\lambda_k) = 0$, this yields $y_i = \bar{y}^0 + \ln(\lambda_i)/A$. An agent's consumption is then equal to the average income plus a state-independent transfer. In general, an agent's consumption is a function of average income depending on Pareto weights and utilities.

3 Arbitrary shocks

3.1 Perfect altruism

We first characterize situations where altruistic transfers generate efficient insurance for every income distribution. Say that agent i is perfectly altruistic towards agent j if $\alpha_{ij} = 1$. The network of perfect altruism is the subnetwork of $\boldsymbol{\alpha}$ which contains perfect altruistic ties. The network of perfect altruism is *strongly connected* if any two agents are connected through a path of perfect altruistic ties. Formally, for any $i \neq j$ there exists a set of l agents $i_1 = i$, i_2,..., $i_l = j$ such that $\forall s < l, \alpha_{i_s i_{s+1}} = 1$. Detailed proofs are provided in the Appendix.

[17] This concerns the extreme parts of the private Pareto frontier. If i is altruistic towards others, the dictatorial private Pareto optimum where $\lambda_j = 0$ if $j \neq i$ is not an altruistic Pareto optimum. In general if $\det(\boldsymbol{\alpha}) \neq 0$, a private Pareto optimum with weights $\boldsymbol{\lambda}$ is an altruistic Pareto optimum iff $(\boldsymbol{\alpha}^t)^{-1}\boldsymbol{\lambda} \gneq \mathbf{0}$. In the literature on welfare evaluation, some researchers argue that social preferences should not be taken into account when evaluating welfare, see e.g. Section 5.4 in Blanchet & Fleurbaey (2006).

Proposition 2 *Informal transfers generate efficient insurance for every income distribution if and only if the network of perfect altruism is strongly connected. In this case, agents have equal Pareto weights.*

To prove sufficiency, we show how to combine equilibrium conditions to obtain the first-order conditions of the planner's program. To prove necessity, we assume that the network of perfect altruism is not strongly connected. We build instances of income distribution for which altruistic transfers do not generate efficient insurance.

Proposition 2 complements earlier results on equal income sharing, see Bloch, Genicot & Ray (2008) and Proposition 1 in Bramoullé & Kranton (2007a).[18] Consider, for example, common utilities and suppose that any altruistic link is perfect $\alpha_{ij} \in \{0, 1\}$. Agent i's best response is to equalize consumption with her poorer friends. Proposition 1 shows that when all agents seek to equalize consumption with their poorer friends and when the altruism network is strongly connected, private transfers necessarily lead to overall equal income sharing, i.e., $y_i = \bar{y}^0$.

This result is straightforward when the network of perfect altruism is complete, as all agents then seek to maximize utilitarian welfare. Proposition 2 shows, however, that perfect altruism also generates efficient insurance in sparse networks such as the star and the line or when two communities are connected by a unique bridge. In these cases, agents' interests are misaligned. Agents care about different subsets of people. Still, under connectedness, the interdependence in individual decisions embedded in equilibrium behavior leads non-cooperative agents to act as if they were following a planner's program.

3.2 Imperfect altruism

We next look at imperfect altruism. In general, how far can informal insurance induced by altruistic transfers move away from efficient insurance with equal Pareto weights? And

[18] Bloch, Genicot & Ray (2008) show that equal sharing in components is the only allocation consistent with the social norm of bilateral equal sharing. Bramoullé & Kranton (2007a) show that if linked pairs meet at random and share income equally, consumption converges to equal sharing in components. By contrast, Proposition 2 identifies conditions under which equal sharing in components emerges as the unique Nash equilibrium of a game of transfers.

how does this depend on the structure of the altruism network?

To answer these questions, we consider common utilities and introduce measures of distance from equal income sharing, as in Ambrus, Mobius & Szeidl (2014). We consider two measures: the average and the largest deviation from the income mean. Formally given income realization $\boldsymbol{y}^0$,

$$\begin{aligned} DISP(\boldsymbol{y}) &= \frac{1}{n}\sum_i |y_i - \bar{y}^0| \\ MDISP(\boldsymbol{y}) &= \max_i |y_i - \bar{y}^0| \end{aligned}$$

Both measures are greater than or equal to zero, and are equal to zero only for equal income sharing. We can then compute their expected value over all income realizations.[19] For instance, $\mathbb{E}DISP(\tilde{\boldsymbol{y}}) = \mathbb{E}\frac{1}{n}\sum_i |y_i - \bar{y}^0|$ such that $\mathbb{E}DISP(\tilde{\boldsymbol{y}}) = 0 \Leftrightarrow \forall \boldsymbol{y}^0, \forall i, y_i = \bar{y}^0$.

Next, we extend the notion of network distance to altruism networks. Following Bourlès, Bramoullé & Perez-Richet (2017), introduce $c_{ij} = -\ln(\alpha_{ij})$ if $\alpha_{ij} > 0$ as the virtual cost of the altruistic link. Stronger links have lower costs. Define the cost of a path as the sum of the costs of the links in the path. If i and j are connected through a path of altruistic links in $\boldsymbol{\alpha}$, define $\hat{c}_{ij}$ as the lowest virtual cost among all paths connecting i to j. For instance when all links have the same strength $\alpha_{ij} \in \{0, \alpha\}$, then $\hat{c}_{ij} = -\ln(\alpha)d_{ij}$ where d_{ij} is the usual network distance between i and j, that is, the length of a shortest path connecting them. When links have different strengths, $\hat{c}_{ij}$ is a measure of altruism distance between i and j accounting for the strength of altruistic ties in indirect paths connecting the two agents. In particular, $\hat{c}_{ij} = 0$ if and only if there is a path of perfect altruistic links connecting i to j.

In our next result, we show that under CARA utilities and for any income realization, distance to equal income sharing is bounded from above by a simple function of network distances, when the altruism network is strongly connected. By contrast, when the altruism

[19] Ambrus, Mobius & Szeidl (2014) consider the Euclidean distance to equal sharing:

$$SDISP(\tilde{\mathbf{y}}) = (\mathbb{E}\frac{1}{n}\sum(y_i - \bar{y}^0)^2)^{1/2}$$

We show below that our result extends to this measure.

network is not strongly connected or under the model of social collateral of Ambrus, Mobius & Szeidl (2014), distance to equal income sharing can take arbitrarily large values.

Proposition 3 *Assume that agents have common CARA utilities. If the altruism network is strongly connected, then for any income realization:*

$$\begin{aligned} DISP(\boldsymbol{y}) &\leq \frac{1}{An^2}\sum_i \max(\sum_j \hat{c}_{ij}, \sum_j \hat{c}_{ji}) \\ MDISP(\boldsymbol{y}) &\leq \frac{1}{An}\max_i(\max(\sum_j \hat{c}_{ij}, \sum_j \hat{c}_{ji})) \end{aligned}$$

If the altruism network is not strongly connected or under the model of social collateral, $\mathbb{E}DISP$ *and* $\mathbb{E}MDISP$ *can take arbitrarily large values.*

We prove the first part of this result by combining, in different ways, inequalities appearing in equilibrium conditions (4) and the second part through examples of income distribution leading to unbounded distances from equal sharing. Note that for CARA utilities, the first part of Proposition 2 directly follows from Proposition 3. The network of perfect altruism is strongly connected if and only if $\forall i, j, \hat{c}_{ij} = 0$, and in that case both bounds are equal to zero.

Proposition 3 identifies specific structural features governing the extent of informal insurance provided by altruistic transfers. It shows, first, that bridges play a critical role. Bridges are links whose removal disconnects the network. To illustrate, consider an altruism network formed of two separate, strongly connected communities. Community-level shocks are not insured, and expected distance from equal sharing can be arbitrarily large. Next, add a single altruistic link between the two communities. Distance from equal sharing is now bounded from above and this bound is independent of the size of the shocks. A large negative shock in one community generates large transfers flowing through the bridge. Both bridge agents play the role of transfer intermediaries and help ensure that informal support from the rich community reaches the poor community.

More generally, Proposition 3 says that the quality of informal insurance depends on distances in the altruism network. To see why, consider links that are undirected and have the same strength: $\alpha_{ij} = \alpha_{ji} \in \{0, \alpha\}$. In that case, the upper bound on $DISP$ is proportional to $\frac{n(n-1)}{n^2}\bar{d}$, where $\bar{d}$ is the average path length in the network, $\bar{d} = \frac{2}{n(n-1)} \sum_{i<j} d_{ij}$. The average deviation from equal income sharing tends to be lower when agents are, on average, closer to each other in the altruism network. Similarly, the upper bound on $MDISP$ is proportional to $\max_i \bar{d}_i$ where $\bar{d}_i$ is the average path length between i and every agent, $\bar{d}_i = (\sum_j d_{ij})/n$. This bound thus depends on the largest average distance between one agent and all other agents in the altruism network. The largest deviation from equal income sharing tends to be lower when this largest average distance is lower.

This notably implies that informal insurance induced by altruism is subject to small-world effects, see Watts & Strogatz (1998). Starting from a spatial network with high average path length, adding a few long-distance connections leads to a disproportionate decrease in average path length and hence to a potentially strong increase in the quality of informal insurance.

Proposition 3 can help empirically distinguish different models of informal transfers in networks. In the model of social collateral, adding a bridge between separate communities has little impact in the presence of large shocks. The reason is that a large negative shock on one community saturates the bridge's capacity constraint, and the distance to equal income sharing can be arbitrarily large.[20] As shown in Proposition 3, this holds more generally. In the model of social collateral, distance from equal sharing is never bounded from above. For any network structure, large shocks can saturate capacity constraints and induce arbitrary large deviations from equal sharing. We next illustrate these effects through numerical simulations.

As a preliminary remark, note that altruistic transfers generally affect all moments of the consumption distribution. Expected consumption may thus differ from expected income. While these redistributive aspects are potentially interesting, we wish to focus here on the risk sharing implications of altruistic transfers. To do so, we identify a natural benchmark where expected consumption is invariant. Altruistic ties are undirected when $\forall i, j, \alpha_{ij} = \alpha_{ji}$. Say that income distribution $\tilde{\boldsymbol{y}}^0$ is symmetric if individuals have the same

[20] Bridges also have little impact on overall informal insurance in a model of risk sharing under local information constraints, see Ambrus, Gao & Milan (2017).

expected income and if the whole profile is distributed symmetrically around its expectation. Formally, $\tilde{\boldsymbol{y}}^0 = \mu\mathbf{1} + \tilde{\boldsymbol{\varepsilon}}$ with $\mathbb{E}(\tilde{\boldsymbol{\varepsilon}}) = \mathbf{0}$ and $f(\boldsymbol{\varepsilon}) = f(-\boldsymbol{\varepsilon})$, where f is the pdf of $\tilde{\boldsymbol{\varepsilon}}$. This covers iid symmetric distributions as well as symmetric distributions with income correlation.

Lemma 1 *Suppose that agents have common CARA utilities, that altruistic ties are undirected, and that income distribution is symmetric. Then* $\forall i, \mathbb{E}y_i = \mathbb{E}y_i^0$.

To prove this result, we prove that if equilibrium transfers $\mathbf{T}$ are associated with shock $\boldsymbol{\varepsilon}$, then reverse transfers $\mathbf{T}^t$ are equilibrium transfers for shock $-\boldsymbol{\varepsilon}$.[21] Symmetry assumptions then guarantee the absence of redistribution in expectations.

We next present results of numerical simulations based on the following assumptions. Agents have CARA utilities $u_i(y) = -e^{-Ay}$ with $A = 0.5$. Incomes are iid binary $y_i^0 = 30 - x$ with probability 0.5 and $y_i^0 = 30 + x$ with probability 0.5. Under altruism, links have strength $\alpha \approx 0.37$ such that $-\ln(\alpha)/A = 2$. Under social collateral, links have capacity constraint $\kappa = 4$ and Pareto weights are equal. We generate $1,000$ realizations of incomes. For each realization, we compute Nash equilibrium transfers and consumption under altruism, and Pareto-constrained transfers and consumption in the social collateral

[21] We thank Adam Szeidl for having first made the connection between this property and the result of no redistribution in expectation.

model.

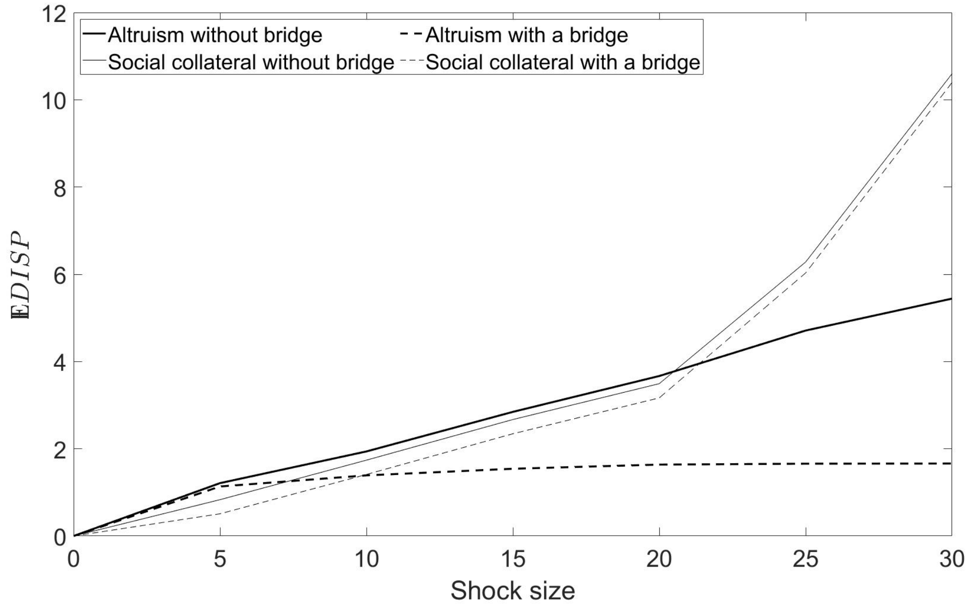

Figure 1. Impact of a bridge: altruism vs social collateral.

Figure 1 depicts the expected deviation from equal income sharing under altruism (in bold) and under social collateral (in light) for two different networks with 20 agents. The first network is composed of two complete, separate subnetworks of 10 agents (continuous lines). In the second network, one bridge is added between these two complete subnetworks (dashed lines). Figure 1 depicts how the distance to efficient insurance varies as the size of the shock x increases, for different models and networks. We see distinct patterns emerging. Under social collateral, expected distance from equal sharing increases with shock size, convexly and without bounds. Adding a bridge improves insurance slightly, but has essentially no impact on the way informal insurance depends on shock size. By contrast, the bridge has a strong impact under altruism. Consistent with Proposition 3, distance from equal sharing quickly reaches an upper bound when the network is connected, but keeps growing when the network is disconnected.

Figure 1 also illustrates the impacts of shock size. Starting from a baseline of similar incomes, small shocks are better insured under social collateral than under altruism. By contrast, large shocks are better insured under altruism than under social collateral. And they are much better insured when the network is connected. We further explore these effects below.

In the Appendix, we show how to extend Proposition 3 in several directions. First, we obtain similar bounds for other measures of distance from equal income sharing. We derive, in particular, a bound for the Euclidean distance $SDISP$, introduced in Ambrus, Mobius & Szeidl (2014). We show that this bound decreases with the average path length and the variance of path lengths in the altruism network. Second, we obtain similar bounds for other utility functions. For common CRRA utilities, we show that $DISP(\boldsymbol{y})/\bar{y}^0$ is bounded from above by a simple function of distances in the altruism network. For common quadratic utilities, a similar property holds for $DISP(\boldsymbol{y})/(y^0_{\max}-\bar{y}^0)$, where $y^0_{\max}$ is the largest possible income value. Third, we obtain different bounds through different arguments. We notably show that when $\boldsymbol{\alpha}$ is strongly connected, $DISP(\boldsymbol{y}) \leq \frac{1}{2A}\max_{i,j}\hat{c}_{ij}$. For undirected binary networks, $\max_{i,j}\hat{c}_{ij} = cd_{\max}$, where $d_{\max}$ is the network's diameter, i.e., the length of the longest shortest path. This improves on Proposition 3 for networks whose diameter is not much greater than their average path length.

While Proposition 3 applies to any income distribution, the bounds' tightness varies. Taking our analysis further, we consider specific assumptions on income shocks. We next look at large shocks on one agent. In Section 4, we characterize what happens for small shocks.

3.3 Large shocks on one agent

We next analyze the impacts of large shocks on one agent, under altruism and under social collateral. Our next result characterizes for both models, how an agent's consumption varies with large income shocks, holding others' incomes fixed.

Proposition 4 *Suppose that agents have common CARA utilities.*

(1) Consider a strongly connected altruism network. Then, $\forall \boldsymbol{y}^0_{-i}$, $\exists Y_H, Y_L$ *such that:*

$y_i^0 \geq Y_H \Rightarrow y_i = \bar{y}^0 + \frac{1}{An}\sum_j \hat{c}_{ij}$ *and* $y_i^0 \leq Y_L \Rightarrow y_i = \bar{y}^0 - \frac{1}{An}\sum_j \hat{c}_{ji}$

(2) Under social collateral, $\forall \boldsymbol{y}_{-i}^0$, $\exists Y_H, Y_L, \kappa_H, \kappa_L$ *such that:*

$y_i^0 \geq Y_H \Rightarrow y_i = y_i^0 - \kappa_H$ *and* $y_i^0 \leq Y_L \Rightarrow y_i = y_i^0 + \kappa_L$

To prove the first part of Proposition (4), we show that if an agent has a large positive shock, money indirectly flows from her to any other agent. If she has a large negative shock, money indirectly flows from any other agent to her. Appropriately combining the Nash conditions then yields the Proposition's formulas. To prove the second part, we rely on the fact that as an agent's realized income increases, efficient insurance demands larger and larger transfers from this agent to the rest of the network, until all the capacity constraints are saturated and no further transfer is possible. Similarly, when the agent's realized income decreases, larger and larger transfers must flow to the agent until all capacity constraints bind.

Proposition (4) implies that, as the size of the income shock grows, the proportion of this shock actually borne by the agent converges to $1/n$ under altruism, as with efficient insurance, while it converges to 1 under social collateral, as in the absence of informal arrangements. Formally, fix $\boldsymbol{y}_{-i}^0$ and assume that agent i's income is $y_i^0 + x$ with $x \geq 0$. Consider, then, $(y_i(x) - y_i(0))/x$ the difference in consumption induced by the shock divided by the size of the shock. A direct application of Proposition (4) shows that $(y_i(x) - y_i(0))/x$ converges to $1/n$ under altruism and to 1 under social collateral as x becomes large. Similarly, if agent i's income is $y_i^0 - x$ then $(y_i(0) - y_i(x))/x$ converges to $1/n$ under altruism and to 1 under social collateral as x becomes large. In this respect, income shocks are asymptotically perfectly insured under altruism and asymptotically uninsured in the social collateral model.

We next illustrate these effects through numerical simulations. Consider the same parameters as above ($A = 0.5$, $\alpha \approx 0.37$, $\kappa = 4$). Consider a line with $n = 10$ agents and i located at one extremity. Assume that $y_i^0 = 50 - x$ and $y_j^0 = 50$ if $j \neq i$. Figure 2 depicts the proportion of the shock borne by the agent $(y_i(0) - y_i(x))/x$ as a function of shock size x under altruism (in bold) and under social collateral (in light). When the shock is very small, transfers lie below capacity constraints and insurance is efficient under social collateral, leading to a proportion of 0.1. By contrast, there is no transfer and no insurance under altruism, and hence a proportion of 1. The situation is quickly reversed as shock size grows. Under social collateral, the agent bears an increasingly large proportion of her shock while under altruism the agent bears a decreasingly small proportion of her shock. In the limit for very large shocks, the outcome is similar to isolation under social collateral and to equal income sharing under altruism.

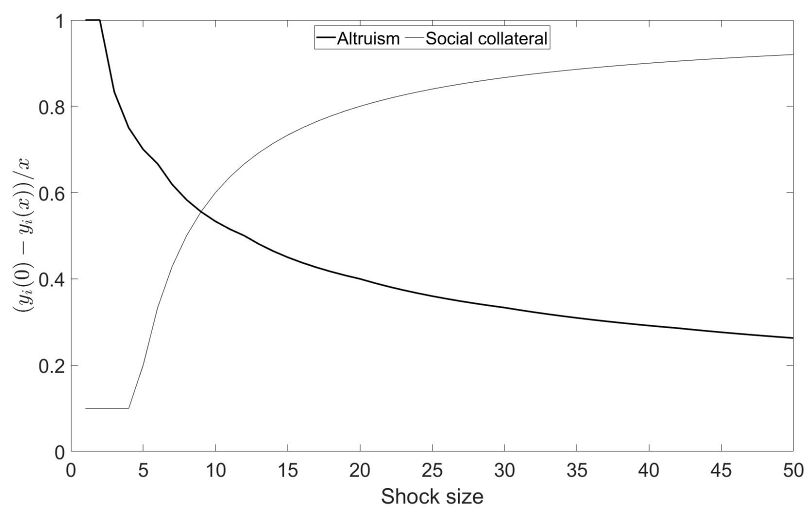

Figure 2. Proportion of a shock borne by an agent: altruism vs social collateral.

Finally, consider stochastic income distributions where a single agent i is subject to large shocks. Consider a binary and undirected network and recall that $\bar{d}_i = (\sum_j d_{ij})/n$ measures the average distance between i and every agent. The arguments in the proof of Proposition 4 can be used to show that under CARA, $DISP(\boldsymbol{y}) = \frac{-\ln(\alpha)}{nA} \sum_j |\bar{d}_i - d_{ij}|$ and $MDISP(\boldsymbol{y}) = \frac{-\ln(\alpha)}{A} \bar{d}_i$ and the largest deviation from equal sharing is reached for agent i. When a single agent is subject to large shocks, the average distance from equal sharing depends on the dispersion of network distances from this agent. And the largest deviation from equal sharing is proportional to the agent's average distance from others.

4 Small shocks

In this section, we characterize what happens for small shocks. More precisely, we consider income shocks that do not affect transfer relationships - who gives to whom. Formally, consider the network of equilibrium transfers $\mathbf{T}$. This network is endogenous and depends on altruistic relations and incomes. In particular, agents only give to others they care about $t_{ij} > 0 \Rightarrow \alpha_{ij} > 0$. Introduce the directed binary graph of transfers $\mathbf{G}$ such that $g_{ij} = 1$ if $t_{ij} > 0$ and $g_{ij} = 0$ if $t_{ij} = 0$. In Bourlès, Bramoullé & Perez-Richet (2017), we showed that generically in $\boldsymbol{\alpha}$ and in $\boldsymbol{y}^0$ there exists $\eta > 0$ such that if $||\hat{\boldsymbol{y}}^0 - \boldsymbol{y}^0|| \leq \eta$ then the unique equilibrium $\hat{\mathbf{T}}$ for incomes $\hat{\boldsymbol{y}}^0$ has the same graph of transfers as the equilibrium $\mathbf{T}$ for incomes $\boldsymbol{y}^0$, and this graph is a forest. Thus, income variations which are relatively small in magnitude generically leave $\boldsymbol{G}$ unchanged.[22] They affect, of course, the amounts transferred and we next characterize the insurance properties of these transfer adjustments.

To present our main result, we introduce some additional notions and notations. A *weak component* of $\mathbf{G}$ is a component of the undirected binary graph where i and j are connected if $g_{ij} = 1$ or $g_{ji} = 1$. When i and j belong to the same weak component of forest graph $\mathbf{G}$, define

$$\bar{c}_{ij} = \sum_{s:g_{i_s i_{s+1}}=1} c_{i_s i_{s+1}} - \sum_{s:g_{i_{s+1} i_s}=1} c_{i_{s+1} i_s}$$

for the unique path $i_1 = i$, i_2,..., $i_l = j$ such that $\forall s, g_{i_s i_{s+1}} = 1$ or $g_{i_{s+1} i_s} = 1$. Note that $\bar{c}_{ij}$ is generally distinct from $\hat{c}_{ij}$. While the altruism distance $\hat{c}_{ij}$ is greater than or equal to zero and only depends on the altruism network $\boldsymbol{\alpha}$, the parameter $\bar{c}_{ij}$ can take negative values and also depends on who gives to whom. The interior of a set is the largest open set included in it. A partition of society is a set of nonempty groups such that every agent belongs to a unique group.

Theorem 1 *(1) Let $\tilde{\boldsymbol{y}}^0$ be an income distribution and* $\mathbf{G}$ *a forest graph such that, for any income realization, there exists a Nash equilibrium of the transfer game with transfer graph* $\mathbf{G}$*. Then altruistic transfers generate efficient insurance within weak components of*

[22] Note that some large income variations also leave $\mathbf{G}$ invariant. For instance with 2 agents and CARA utilities, i gives to j in equilibrium iff $y_i^0 \geq y_j^0 + c_{ij}/A$.

G. *If agent i belongs to weak component C of size n_C, his Pareto weight λ_i is such that $\ln(\lambda_i) = (\sum_{j \in C} \bar{c}_{ij})/n_C$ under normalization $\sum_{j \in C} \ln(\lambda_j) = 0$.*
(2) Consider an income distribution whose support's interior is non-empty. Suppose that there is a partition of society such that altruistic transfers generate efficient insurance within groups, Then, generically in $\boldsymbol{\alpha}$, the graph of transfers is constant across income realizations in the support's interior and these groups correspond to the weak components of the transfer graph.

To prove the first part of Theorem 1, we compare equilibrium conditions with the first-order conditions of the planner's program. When i makes transfers to j in equilibrium, the ratio of their marginal utilities is equal to the altruistic coefficient: $u_i'(y_i)/u_j'(y_j) = \alpha_{ij}$. Under efficient insurance, we would have $u_i'(y_i)/u_j'(y_j) = \lambda_j/\lambda_i$. We thus look for Pareto weights such that $\lambda_j/\lambda_i = \alpha_{ij}$. Naturally, this equality cannot generally be satisfied for all pairs of agents. We show in the Appendix how to exploit the forest structure of equilibrium transfers to find appropriate Pareto weights. Our proof is constructive and based on the explicit formulas provided in the Theorem. Note that the Pareto weights only depend on $\boldsymbol{\alpha}$ and **G** and hence do not depend on the specific income realization. Since money flows within but not between weak components, this leads to efficient insurance within weak components.

In the second part of Theorem 1, we show that shocks leaving the structure of giving relationships unchanged are, generically, the only situations where altruistic transfers generate constrained efficient insurance. We provide a sketch of the proof here. The main idea is to exploit the first part of the Theorem: locally around some income profile, altruistic transfers yield constrained Pareto efficiency with known features (groups and Pareto weights). These features must then be consistent with the original assumed pattern of constrained efficiency, and we show that this can only happen when the graph of transfers is invariant. An important step in the proof is to show that generically in $\boldsymbol{\alpha}$, the Pareto weight mapping $\mathbf{G} \to \boldsymbol{\lambda}(\mathbf{G})$ is injective. Overall, this result provides a generic characterization of situations where there is constrained efficient insurance.

The first part of Theorem 1 extends Theorem 3 in Bourlès, Bramoullé & Perez-Richet

(2017). It characterizes the income sharing functions uncovered in that result and shows that the weak components of the transfer graphs form endogenous risk sharing communities.

Theorem 1 shows that, following small shocks, adjustments in altruistic transfers satisfy a property of constrained efficiency. Within a weak component of $\mathbf{G}$, agents act as if they were following a planner's program. The quality of informal insurance provided by altruistic transfers then depends on the connectivity of the transfer graph. Informal insurance is efficient if $\mathbf{G}$ is weakly connected. This happens, for instance, when one agent is much richer than all other agents. By contrast, agents fully support their income risks when $\mathbf{G}$ is empty. This happens when $\forall i, j, \alpha_{ij} < 1$ and $\tilde{\boldsymbol{y}} = \tilde{y}^0\mathbf{1} + \tilde{\boldsymbol{\varepsilon}}$ for small enough $\tilde{\boldsymbol{\varepsilon}}$. When income differences among agents are small in all realizations, agents make no altruistic tranfers in equilibrium. By contrast, such small shocks would be efficiently insured in the social collateral model.

More generally, the extent of informal insurance depends on the number and sizes of $\mathbf{G}$'s weak components. Under common CARA utilities, the equilibrium consumption of agent i in component C is equal to $y_i = \bar{y}^0_C + \ln(\lambda_i)/A$. Under iid income shocks, this implies that $Var(y_i) = Var(y_i^0)/n_C$ and an increase in components' sizes leads to a decrease in consumption variance for all agents.[23]

The Pareto weights capture how agents' private preferences are represented in the equivalent planner's program. They reflect agents' positions in the graph of transfers and depend on the graph's full structure. For instance, a giving line where $t_{i_1 i_2} > 0$, $t_{i_2 i_3} > 0$, ..., $t_{i_{n-1} i_n} > 0$ yields $\lambda_1 > \lambda_2 > ... > \lambda_n$. More generally an agent's preferences tend to be well-represented when this agent has a relatively "higher" position in the network of transfers. This happens when she tends to give to others towards whom she is not too altruistic, inducing higher c's.

A further implication is that local changes may have far-reaching consequences. Suppose, for instance, that $g_{ij} = 1$ in forest graph $\mathbf{G}$ and consider a small increase in α_{ij} that

[23] Effects are more complex when shocks are not iid. When shocks are independent but not identical, $Var(y_i) = (\sum_{j\in C} Var(y_j^0))/n_C^2$. Consumption variance may be greater than income variance for an agent with relatively low income variance. However, $\sum_{i\in C} Var(y_i) = (\sum_{i\in C} Var(y_i^0))/n_C < \sum_{i\in C} Var(y_i^0)$. Increases in variance for some agents are more than compensated by decreases in variance for others.

does not change the pattern of giving relationships. Let C be the weak component of i and j and define C_i as the weak component of i in the graph obtained from $\mathbf{G}$ by removing the link ij, and similarly for C_j. Note that $C = C_i \cup C_j$ and $C_i \cap C_j = \varnothing$. Informally, C_i represents agents indirectly connected to the giver while C_j represents agents indirectly connected to the receiver.

Proposition 5 *Suppose that the binary graph of transfers* $\mathbf{G}$ *is a forest graph and that* $g_{ij} = 1$. *Consider a small increase in* α_{ij} *leaving* $\mathbf{G}$ *unaffected. Then,* λ_k *decreases if* $k \in C_i$ *and increases if* $k \in C_j$.

Therefore the normalized Pareto weights of the giver and of agents indirectly connected to her decrease, while the normalized Pareto weights of the receiver and of agents indirectly connected to her increase. This implies that the consumption of agents in C_i decreases while the consumption of agents in C_j increases, and hence Proposition 5 extends the first part of Theorem 4 in Bourlès, Bramoullé & Perez-Richet (2017).

5 Network structure and informal insurance

In this Section, we study the impact of the network structure on consumption smoothing. How is the position of an agent in the altruism network related to her consumption variance? How do altruistic transfers affect the correlation structure of consumption streams across individuals? How does a new link between two agents affect their consumption variance? How does it affect the consumption variance of other agents in the network? We uncover some complex effects, which we analyze through a combination of analytical results and numerical simulations.

We present results of numerical simulations based on the following parameter values. We consider a real network of informal lending and borrowing relationships, connecting 111 households in a village in rural India drawn from the data analyzed in Banerjee et al. (2013). The network is depicted in Figure 1. Altruistic links have strength α and agents have CARA utilities $u_i(y) = -e^{-Ay}$ with $-\ln(\alpha)/A = 3$. Incomes are iid binary: $y_i^0 = 0$ with probability 0.5 and 20 with probability 0.5. We consider 10,000 realizations of incomes and, for each realization, we compute equilibrium transfers and consumption. The analysis was replicated with lognormal incomes with the same mean and variance, and all the results reported below were found to be qualitatively robust.

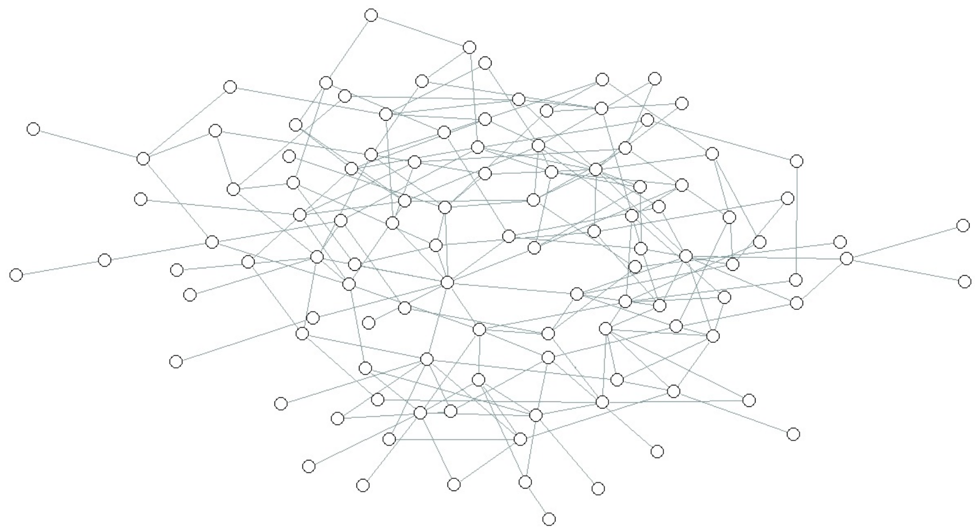

Figure 3. A Network of Informal Risk Sharing

We start by looking at the relationship between the network structure and the consumption variance - covariance matrix. Are more central agents better insured? We compute correlation coefficients between consumption variance and different measures of centrality (degree, betweenness centrality, eigenvector centrality), see Table 1. Correlation is clearly negative and both quantitatively and statistically significant.

Table 1. Correlation between Centralities and Consumption Variance

	Variance
Degree	-0.7657***
Between	-0.5148***
Eigen	-0.6800***

***denotes statistical significance at the 1% level

Simulation Result 1: *More central agents tend to have lower consumption variance.*

On this dimension, the model of altruism in networks generates predictions similar to those of the model of social collateral. Its predictions differ from those of the model of local information constraints, which generates positive correlation between consumption variance and centrality, see Ambrus, Gao & Milan (2017). Under local information constraints, more central agents act as quasi insurance providers for more peripheral neighbors: they bear a larger share of risk and are compensated by higher state-independent transfers.

We next look at correlations in consumption streams across individuals. We show that, starting from independent incomes, altruistic transfers necessarily induce weakly positive covariance in consumption across agents. This holds for any pair of agents, any altruism network and any utility functions.

Proposition 6 *Suppose that incomes are independent across agents.* $\forall i, j, cov(\tilde{y}_i, \tilde{y}_j) \geq 0$.

We obtain this result by relying on the global comparative statics of consumption with respect to incomes, see Theorem 3 in Bourlès, Bramoullé & Perez-Richet (2017). This result says that y_i is weakly increasing in y_j^0 for any i, j. A positive shock on any agent's income thus induces weakly positive changes in the consumption of every agent in society, and conversely for negative shocks. To prove the result, we then combine this property with classical properties of the covariance operator.

Altruistic transfers thus tend to generate positive correlation across individuals' consumption streams. We next explore through simulations how these correlations depend on the network distance between agents. Figure 2 depicts the correlogram of consumption correlation between y_i and y_j as a function of network distance between i and j. We consider all pairs at given distance d and compute the average correlation coefficient (plain line) as well as the 5th and 95th percentiles of the correlation distribution (dashed lines). We see that consumption correlation is generally positive, consistently with Proposition 6. Furthermore,

Simulation Result 2: *Consumption correlation tends to decrease with network distance.*

Consumption correlation can reach very high levels for direct neighbors and then tends to decrease at a decreasing rate as network distance increases.

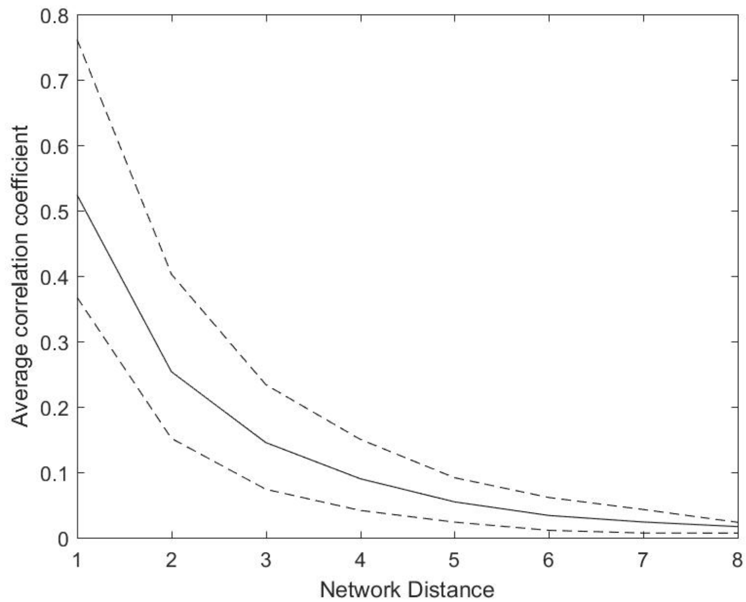

Figure 4. Consumption Correlation as a Function of Network Distance

Finally, we study the impact of adding one altruistic link on agents' consumption variances. We ran extensive numerical simulations for a variety of income distributions and network structures. With iid incomes and under the assumptions underlying Lemma 1, the consumption variance of the two agents becoming connected generally drops.[24] This is consistent with Simulation Result 1: acquiring more links, or a better position, in the altruism network allows agents to reduce consumption variability. By contrast, the new link may increase or decrease the consumption variance of other agents in the network. Two opposite forces are at play here. On the one hand, the new link provides a source of additional indirect support, which can help further smooth consumption. On the other hand, the new neighbor is also a competitor for the support of the existing neighbor, which can reduce consumption smoothing.

For instance, with 3 agents, iid binary incomes and CARA utilities, we can show the

[24] We provide a simple example in the Appendix showing that if incomes are correlated, obtaining a new connection may lead to an increase in consumption variance.

following result (proof in Appendix). Start from a situation where agent 1 is connected to agent 2 but not to agent 3. Add the connection between 2 and 3 to form a line, and $Var(y_1)$ drops. Next, close the triangle by adding the connection between 1 and 3, and $Var(y_2)$ increases. Connecting the two peripheral agents of a 3-agent line leads to an increase in consumption variance for the center. Consider, next, a line connecting 6 agents, labeled 1 to 6 and with agents 1 and 6 at the periphery. Add the link between 1 and 6, transforming the line in a circle. Numerical simulations show that consumption variance decreases for agents 1 and 6 and for their direct neighbors, agents 2 and 5. By contrast, consumption variance increases for neighbors' neighbors, agents 3 and 4.

Finally, we look at the impact of adding a link to a complex, real-world network, as shown in Figure 3. We depict the new link in bold and focus on the region of the network close to the new link. No change in variance is detected outside this region. Nodes for which we detect a change in consumption variance are depicted in grey, with a symbol describing the direction of the change.[25] We observe both increases and decreases in consumption variance for indirect neighbors. To sum up,

Simulation Result 3: *Connecting two agents generally leads to a decrease in their consumption variance and can lead to a decrease or an increase in the consumption variance of indirect neighbors.*

[25] Because of numerical variability, we set a relatively high detection threshold t and only report variance changes $\Delta Var(y_i)$ such that $|\Delta Var(y_i)| \geq t$. Thus, Figure 3 likely does not report false positives (detected changes are likely true changes) and may report false negatives (some true changes may not be detected).

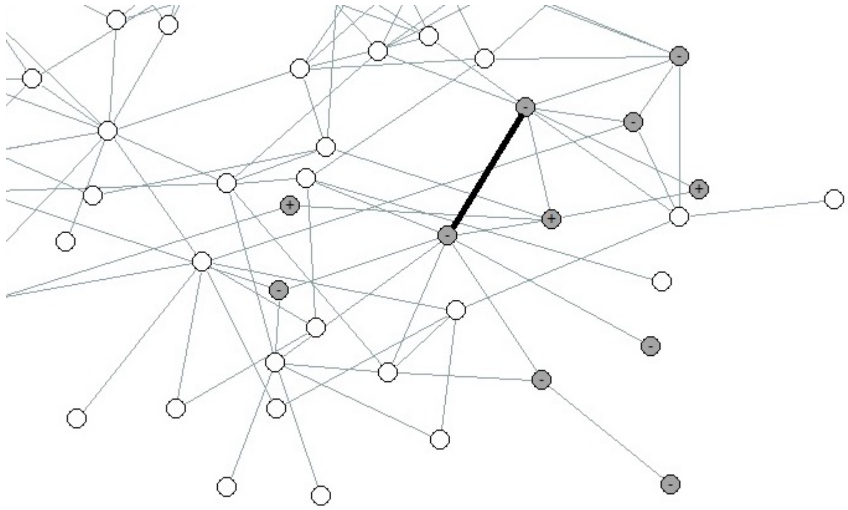

Figure 5. Impact of a New Link on Consumption Variances

6 Discussion and Conclusion

In reality, informal transfers are likely explained by a combination of altruism, networks, and informal insurance contracts. Incorporating altruism networks in dynamic models of insurance contracts with frictions and bringing these extended models to data are important and challenging objectives for future research.[26]

Many studies of informal insurance contracts with frictions rely on models with two agents. Empirical implementations on groups with n agents generally make (strong) assumptions and approximations to avoid the curse of dimensionality.[27] With an altruism network, avoiding this curse will be difficult. Even when agents have common private utilities, different network positions induce different altruistic utility functions.

In a model of limited commitment, altruism affects the incentive compatibility con-

[26] Another important direction for future research is to analyze the interaction between formal and informal risk sharing institutions.

[27] Following Ligon, Thomas & Worrall (2002), many studies assume common CRRA utility functions and do not consider the constrained efficient risk sharing contracts among n households. Rather, they consider the simpler problem of an agent sharing risk with another agent representing the rest of the group, under the (incorrect) assumption of unconstrained efficient risk sharing among these $n-1$ households, e.g. Dubois, Jullien & Magnac (2008), Laczó (2015). As observed in Ábrahám & Laczó (2017, p.25), "We do not know of a satisfactory treatment of the N-household case in limited commitment models, and structural empirical studies of risk sharing in village economies follow the household versus rest of the village approximation."

straint in important ways. Recall that this constraint says that for every agent, the current utility of making contractual transfers plus the expected future utility from the contract should be at least as high as the current utility of reneging plus the expected future utility from punishment. Under altruism, agents care about others and take into account their transfers' impact on others.[28] Altruism thus has two opposite effects on incentives to share risk. On the one hand, altruism reduces the temptation to deviate, since agents who make transfers partly internalize others' benefits from receiving these transfers. On the other hand, altruism can reduce the effectiveness of punishments, a version of the Samaritan's dilemma. Note that reversion to autarky is not necessarily natural when agents care about each other. In a two-agent model, Foster and Rosenzweig (2001) consider Nash reversion instead. They assume that if one agent does not meet his transfer obligations, the two agents then play a static Nash equilibrium in every subsequent period. As we showed in our analysis, the risk sharing generated in a static Nash equilibrium can actually be quite extensive. Thus, agents may face less severe punishments in a network with strong altruistic ties. The overall impact of altruism networks on the risk sharing attainable under limited commitment is, a priori, ambiguous. This deserves further investigation.[29]

To conclude, we analyze the risk sharing implications of altruism networks, in the absence of formal or informal insurance contracts. We find that altruistic transfers have a first-order impact on risk. Altruistic transfers generate efficient insurance for any income distribution when the network of perfect altruistic ties is strongly connected. More generally, the average and largest deviation from the income mean tend to increase with the average path length of the altruism network. Bridges can deeply alter the overall transfer patterns and can generate good overall risk sharing. Large shocks on one agent are well-insured in connected altruism networks. These distinct predictions can help empirically identify the motives behind informal transfers. We further show that when income

[28]From a technical point of view, the derivative of the Lagrangian with respect to the utility promise made to agent i in some state of the world then depends on the Lagrange multipliers of the incentive compatibility constraints of others who care about i.

[29]An interesting particular case is when agents have some altruistic links, for instance towards kin or members of the same caste, and also have formal or informal contractual relationships with agents they do not care about, such as labor relationships with members of others castes.

shocks leave the structure of giving relationships unchanged, altruistic transfers generate constrained efficient risk sharing within the weak components of the transfer network. Conversely, these are generically the only income shocks where constrained efficiency holds. Finally, we uncover and investigate complex structural effects.

APPENDIX

Extension of previous results to perfect altruism. Bourlès, Bramoullé & Perez-Richet (2017) assume that $\alpha_{ij} < 1$ and $u_i'(y) > \alpha_{ij} u_j'(y)$. We relax these assumptions slightly here by assuming that $\alpha_{ij} \leq 1$ and $u_i'(y) \geq \alpha_{ij} u_j'(y)$, allowing for perfect altruism. Perfect altruism gives rise to unbounded Nash equilibria caused by cycles in transfers. For instance if two agents are perfectly altruistic towards each other $\alpha_{12} = \alpha_{21} = 1$ and have the same utility functions and incomes, Nash equilibria are transfer profiles of the form $(t_{12} = t, t_{21} = t)$, leaving consumption unaffected. Theorems 1-4 in Bourlès, Bramoullé & Perez-Richet (2017) still hold under the new assumptions with two caveats. (1) Equilibrium transfers are now not necessarily acyclic. An acyclic Nash equilibrium still exists, however. To see why, suppose that there is a cycle in transfers: $t_{i_1 i_2} > 0$,..., $t_{i_l i_1} > 0$. This implies that $u_{i_1}'(y_{i_1})/u_{i_2}'(y_{i_2}) = \alpha_{i_1 i_2}$,...,$u_{i_l}'(y_{i_l})/u_{i_1}'(y_{i_1}) = \alpha_{i_l i_1}$. Multiplying all equalities yields $1 = \alpha_{i_1 i_2} ... \alpha_{i_l i_1}$ and hence $\alpha_{i_l i_1} = ... = \alpha_{i_l i_1} = 1$. Cycles in transfers can only happen in cycles of perfect altruistic ties. Then, let $t = \min(t_{i_1 i_2}, ..., t_{i_l i_1})$. Removing t from all transfers in the cycle yields another Nash equilibrium, and repeating this operation leads to an acyclic Nash equilibrium. (2) The genericity condition in $\boldsymbol{\alpha}$ must be supplemented by the condition that $\boldsymbol{\alpha}$ does not contain directed cycles of perfect altruistic ties. This then guarantees that Nash equilibria are acyclic.

Proof of Proposition 2. We will make use of the following properties established in Bourlès, Bramoullé & Perez-Richet (2017). Define $\hat{\alpha}_{ij} = e^{-\hat{c}_{ij}}$ if $\hat{c}_{ij} < \infty$ and $\hat{\alpha}_{ij} = 0$ otherwise. Then, $\forall i, j, u_i'(y_i) \geq \hat{\alpha}_{ij} u_j'(y_j)$ and $u_i'(y_i) = \hat{\alpha}_{ij} u_j'(y_j)$ if there is a directed path connecting i to j in $\mathbf{T}$. Next, suppose that i is much richer than everyone else. Then money indirectly flows from i to every other agent j such that $\hat{\alpha}_{ij} > 0$ and $\forall i, j : \hat{\alpha}_{ij} > 0, u_i'(y_i) = \hat{\alpha}_{ij} u_j'(y_j)$.

Observe that the network of perfect altruistic ties is strongly connected if and only if $\forall i, j, \hat{\alpha}_{ij} = 1$. If this holds, then $\forall i, j, u_i'(y_i) \geq u_j'(y_j)$ and hence $u_i'(y_i) = u_j'(y_j)$. These are the first-order conditions of the problem of maximizing utilitarian welfare. Next, suppose that there exist i and j such that $\hat{\alpha}_{ij} < 1$. Define $\boldsymbol{y}^0$ such that $y_i^0 = Y$ and $\forall k \neq i, y_k^0 = 0$. If $\hat{\alpha}_{ij} = 0$, money cannot flow from i to j. As Y increases, consumption y_i tends to ∞ while $y_j = 0$. If Y is large enough, $u_i'(y_i) < u_j'(y_j)$. If $\hat{\alpha}_{ij} > 0$, then $u_i'(y_i) = \hat{\alpha}_{ij} u_j'(y_j) < u_j'(y_j)$ if Y is large enough. Similarly, define $\tilde{\boldsymbol{y}}^0$ such that $\tilde{y}_j^0 = Y$ and $\forall k \neq j, \tilde{y}_k^0 = 0$. Since $\hat{\alpha}_{ji} \leq 1$, $u_j'(\tilde{y}_j) \leq u_i'(\tilde{y}_i)$ if Y large enough. Under efficient insurance, we would then have $\lambda_j < \lambda_i$ and $\lambda_j \geq \lambda_i$, a contradiction. Therefore, altruistic transfers do not generate efficient insurance. QED.

Proof of Proposition 3. Recall; $\forall i, j, u_i'(y_i) \geq \hat{\alpha}_{ij} u_j'(y_j)$. This is equivalent to: $(u_j')^{-1}(\frac{1}{\hat{\alpha}_{ij}} u_i'(y_i)) \leq y_j$. Summing over j leads to:

$$\sum_j (u_j')^{-1}(\frac{1}{\hat{\alpha}_{ij}} u_i'(y_i)) \leq n\bar{y}^0$$

We also have $\forall i,j, u'_j(y_j) \geq \hat{\alpha}_{ji} u'_i(y_i)$ and hence $y_j \leq (u'_j)^{-1}(\hat{\alpha}_{ji} u'_i(y_i))$, leading to

$$n\bar{y}^0 \leq \sum_j (u'_j)^{-1}(\hat{\alpha}_{ji} u'_i(y_i))$$

Under common CARA utilities, this yields $-\frac{1}{An}\sum_j \hat{c}_{ji} \leq y_i - \bar{y}^0 \leq \frac{1}{An}\sum_j \hat{c}_{ij}$ and hence $|y_i - \bar{y}^0| \leq \frac{1}{An}\max(\sum_j \hat{c}_{ij}, \sum_j \hat{c}_{ji})$. Finally, $DISP(\boldsymbol{y}) \leq \frac{1}{An^2}\sum_i \max(\sum_j \hat{c}_{ij}, \sum_j \hat{c}_{ji})$.

Next, we compute similar bounds for other measures of distance and other utility functions. Introduce $SDISP(\tilde{\boldsymbol{y}}) = [\mathbb{E}\frac{1}{n}\sum_i (y_i - \bar{y}^0)^2]^{1/2}$ as in Ambrus, Mobius & Szeidl (2014). We obtain:

$$SDISP(\boldsymbol{y}) \leq \frac{1}{A}\frac{1}{n^{3/2}}[\sum_i \max(\sum_j \hat{c}_{ij}, \sum_j \hat{c}_{ji})^2]^{1/2}$$

When the network is binary and undirected, the bound becomes $\frac{-\ln(\alpha)}{A}\frac{1}{n^{3/2}}[\sum_{i,j} d_{ij}^2]^{1/2}$. Then, $\frac{1}{n(n-1)}\sum_{i,j} d_{ij}^2 = \bar{d}^2 + V(\mathbf{d})$ where $V(\mathbf{d})$ is the variance of path lengths. Thus, $SDISP$ tends to be lower when average path length and path length variance are lower.

Alternatively, consider common CRRA utilities: $u(y) = y^{1-\gamma}/(1-\gamma)$ if $\gamma \neq 1$, $\gamma > 0$ and $u(y) = \ln(y)$ if $\gamma = 1$. This yields

$$(\frac{n}{\sum_j \hat{\alpha}_{ji}^{-1/\gamma}} - 1)\bar{y}^0 \leq y_i - \bar{y}^0 \leq (\frac{n}{\sum_j \hat{\alpha}_{ij}^{1/\gamma}} - 1)\bar{y}^0$$

and hence

$$DISP(\boldsymbol{y}) \leq \frac{1}{n}\sum_i \max(1 - \frac{n}{\sum_j \hat{\alpha}_{ji}^{-1/\gamma}}, \frac{n}{\sum_j \hat{\alpha}_{ij}^{1/\gamma}} - 1)\bar{y}^0$$

Next, consider common quadratic utilities: $u(y) = y - \frac{1}{2}\lambda y^2$, under the assumption that $y \leq y_{\max}^0 = 1/\lambda$. A similar reasoning yields

$$(1 - \frac{n}{\sum_j \hat{\alpha}_{ji}})(y_{\max}^0 - \bar{y}^0) \leq y_i - \bar{y}^0 \leq (1 - \frac{n}{\sum_j \frac{1}{\hat{\alpha}_{ij}}})(y_{\max}^0 - \bar{y}^0)$$

and hence

$$DISP(\boldsymbol{y}) \leq \frac{1}{n}\sum_i \max(\frac{n}{\sum_j \hat{\alpha}_{ji}} - 1, 1 - \frac{n}{\sum_j \frac{1}{\hat{\alpha}_{ij}}})(y_{\max}^0 - \bar{y}^0)$$

Next, consider common CARA utilities and suppose that the network of altruism is not strongly connected. Then, there exists a set S such that $S \neq \varnothing$, $N - S \neq \varnothing$, there exists a path between any two agents in S in $\boldsymbol{\alpha}$, and no agent in S cares about an agent not in S. Consider the income distribution such that $y_i^0 = Y > 0$ if $i \in S$ and $y_i^0 = 0$ if $i \notin S$. Then, there is no transfer in equilibrium and $\boldsymbol{y} = \boldsymbol{y}^0$. Here, $\bar{y}^0 = \frac{n_S}{n}Y$ and $|y_i - \bar{y}^0| = \frac{n-n_S}{n}Y$ if $i \in S$ and $\frac{n_S}{n}Y$ if $i \notin S$. This yields $DISP(\boldsymbol{y}) = \mathbb{E}DISP = 2\frac{n_S(n-n_S)}{n}Y$ and $MDISP(\boldsymbol{y}) = \mathbb{E}MDISP = \frac{\max(n_S, n-n_S)}{n}Y$ and both distances becomes arbitrarily large as Y increases.

Finally, consider the social collateral model and the income distribution such that

$y^0_{i_0} = Y > 0$ and $y^0_i = 0$ if $i \neq i_0$. Proposition 4 shows that there exist Y_H and κ_H such that $Y \geq Y_H \Rightarrow y_{i_0} = Y - \kappa_H$. Moreover, since the only source of income is i_0, we then have $y_i \leq \kappa_H$ if $i \neq i_0$. Here, $\bar{y}^0 = \frac{1}{n}Y$ and $|y_{i_0} - \bar{y}^0| = \frac{n-1}{n}Y - \kappa_H$ while $|y_i - \bar{y}^0| \geq \frac{1}{n}Y - \kappa_H$ if Y is large enough. Therefore, $DISP(\boldsymbol{y}) = \mathbb{E}DISP \geq \frac{2(n-1)}{n^2}Y - \kappa_H$ and $MDISP(\boldsymbol{y}) = \mathbb{E}MDISP \geq \frac{1}{n}Y - \kappa_H$ and both distances become arbitrarily large as Y increases. QED.

Proof of Lemma 1. We first establish that reverse transfers form an equilibrium for the opposite shock. Denote by $\boldsymbol{y}^0(\boldsymbol{\varepsilon}) = \mu\mathbf{1} + \boldsymbol{\varepsilon}$ and by $\boldsymbol{y}(\boldsymbol{\varepsilon})$ the associated equilibrium consumption. Let $\mathbf{T}$ be a Nash equilibrium for incomes $\mathbf{y}^0(\boldsymbol{\varepsilon})$ leading to consumption $\mathbf{y}(\boldsymbol{\varepsilon})$. We now show that $\mathbf{T}^t$ is a Nash equilibrium for incomes $\mathbf{y}^0(-\boldsymbol{\varepsilon})$ and $\mathbf{y}(\boldsymbol{\varepsilon}) - \mathbf{y}^0(\boldsymbol{\varepsilon}) = \mathbf{y}^0(-\boldsymbol{\varepsilon}) - \mathbf{y}(-\boldsymbol{\varepsilon})$. To see why, note that $\boldsymbol{y} = \mu\mathbf{1} + \boldsymbol{\varepsilon} - \mathbf{T1} + \mathbf{T}^t\mathbf{1}$. Denote by $\boldsymbol{y}'$ the consumption associated with transfers $\mathbf{T}^t$ when incomes are $\mu\mathbf{1} - \boldsymbol{\varepsilon}$. Then, $\boldsymbol{y}' = \mu\mathbf{1} - \boldsymbol{\varepsilon} - \mathbf{T}^t\mathbf{1} + \mathbf{T1}$. Comparing yields: $\boldsymbol{y} - \mu\mathbf{1} - \boldsymbol{\varepsilon} = \mu\mathbf{1} - \boldsymbol{\varepsilon} - \boldsymbol{y}'$ and hence $\boldsymbol{y}(\varepsilon) - \boldsymbol{y}^0(\varepsilon) = \boldsymbol{y}^0(-\varepsilon) - \boldsymbol{y}'$. Equilibrium conditions on $\mathbf{T}$ are: (1) $\forall i, j, y_i - y_j \leq c_{ij}/A$, and (2) $t_{ij} > 0 \Rightarrow y_i - y_j = c_{ij}/A$. Let us check that $\mathbf{T}^t$ satisfy the equilibrium conditions for incomes $\boldsymbol{y}^0(-\boldsymbol{\varepsilon})$. We have: $y'_i = 2\mu - y_i$. This implies that $y'_i - y'_j = y_j - y_i$. Therefore, $\forall i, j, y'_i - y'_j = y_j - y_i \leq c_{ji}/A = c_{ij}/A$ since the ties are undirected. In addition, $(\mathbf{T}^t)_{ij} = t_{ji}$ and $t_{ji} > 0 \Rightarrow y_j - y_i = c_{ji}/A \Rightarrow y'_i - y'_j = c_{ij}/A$.

We have: $\mathbb{E}(y_i - y^0_i) = \int_{\boldsymbol{\varepsilon}}[y_i(\boldsymbol{\varepsilon}) - y^0_i(\boldsymbol{\varepsilon})]f(\boldsymbol{\varepsilon})d\boldsymbol{\varepsilon}$. In the integral, the term associated with no shock is equal to 0, $y_i(\mathbf{0}) = y^0_i(\mathbf{0})$. The term associated with shock $\boldsymbol{\varepsilon}$ is equal to $[y_i(\boldsymbol{\varepsilon}) - y^0_i(\boldsymbol{\varepsilon})]f(\boldsymbol{\varepsilon})d\boldsymbol{\varepsilon}$. The term associated with shock $-\boldsymbol{\varepsilon}$ is equal to $[y_i(-\boldsymbol{\varepsilon}) - y^0_i(-\boldsymbol{\varepsilon})]f(-\boldsymbol{\varepsilon})d\boldsymbol{\varepsilon} = [y^0_i(\boldsymbol{\varepsilon}) - y_i(\boldsymbol{\varepsilon})]f(\boldsymbol{\varepsilon})d\boldsymbol{\varepsilon}$ by Lemma A4 and by shock symmetry. The sum of these terms is then equal to 0 and the integral aggregates such sums. QED.

Proof of alternative bound on p.15. Denote by $\hat{c}_{\max} = \max_{i,j} \hat{c}_{ij}$. Since $\forall i, j, u'_i(y_i) \geq \hat{\alpha}_{ij}u'_j(y_j)$, $\forall i, j, y_i \leq y_j + \hat{c}_{ij}/A \leq y_j + \hat{c}_{\max}/A$. This implies that $y_{\max} - y_{\min} \leq \hat{c}_{\max}/A$ where $y_{\max} = \max_i y_i$ and $y_{\min} = \min_i y_i$. Consider the problem of maximizing $DISP(\boldsymbol{y})$ under the constraint that $y_{\max} - y_{\min} = \Delta$ where Δ is some arbitrarily fixed value. The solution to this problem is to set $y_i = y_{\max}$ for $n/2$ agents if n is even and for $(n+1)/2$ agents if n is odd and $y_i = y_{\min}$ for $n/2$ agents if n is even and for $(n-1)/2$ agents if n is odd. This yields $DISP(\boldsymbol{y}) = \frac{1}{2}\Delta$ if n is even and $= (\frac{1}{2} - \frac{1}{2n^2})\Delta$ if n is odd. This implies that, in general, $DISP(\boldsymbol{y}) \leq \frac{1}{2}(y_{\max} - y_{\min}) \leq \frac{1}{2}\hat{c}_{\max}/A$. QED.

Proof of computations on p.11. Suppose agent i is subject to large shocks. If the shock is positive, $\forall j, u'_i(y_i) = \hat{\alpha}_{ij}u'_j(y_j)$. This yields $y_i = y_j + \frac{c}{A}d_{ij}$. Taking the average over j yields $y_i = \bar{y}^0 + \frac{c}{A}\bar{d}_i$ and $y_j = \bar{y}^0 + \frac{c}{A}(\bar{d}_i - d_{ij})$. If the shock is negative, $\forall j, u'_j(y_j) = \hat{\alpha}_{ji}u'_i(y_i)$ and hence $y_j = y_i + \frac{c}{A}d_{ij}$ and $y_i = \bar{y}^0 - \frac{c}{A}\bar{d}_i$ and $y_j = \bar{y}^0 + \frac{c}{A}(d_{ij} - \bar{d}_i)$. This leads to $\frac{1}{n}\sum_i |y_i - \bar{y}^0| = \frac{c}{nA}\sum_i |\bar{d}_i - d_{ij}|$. QED.

Proof of Proposition 4. (1) We work with the matrix of lowest virtual costs $\hat{c}_{ij}$, as we showed in Bourlès, Bramoullé & Perez-Richet (2017) that it yields the same equilibrium consumption as the original problem. Let

$$Y_H = \max_{j \neq i} \sum_{k \neq i} \left\{ \left(y_j^0 + \frac{1}{A} \hat{c}_{ij} \right) - \left(y_k^0 + \frac{1}{A} \hat{c}_{ik} \right) \right\},$$

and suppose $y_i^0 \geq Y_H$. Then consider the candidate equilibrium transfers where, for all $j \neq i$:

$$t_{ij} = \frac{1}{n} \sum_k \left\{ \left(y_k^0 + \frac{1}{A} \hat{c}_{ik} \right) - \left(y_j^0 + \frac{1}{A} \hat{c}_{ij} \right) \right\}$$

and other transfers are 0. $y_i^0 \geq Y_H$ implies that each t_{ij} is nonnegative. These transfers imply that $y_i = \bar{y}^0 + \frac{1}{An} \sum_j \hat{c}_{ij}$, as in the proposition. Furthermore, we have, for every $j \neq i$,

$$y_i - y_j = \frac{1}{A} \hat{c}_{ij},$$

and, for every $j, k \neq i$,

$$y_j - y_k = \frac{1}{A} \left(\hat{c}_{ik} - \hat{c}_{ij} \right) \leq \frac{1}{A} \hat{c}_{jk}.$$

Therefore, the first-order conditions are satisfied and these transfers indeed form an equilibrium. The low income case can be shown in a similar way.

(2) Consider a fixed vector of Pareto weights $\boldsymbol{\lambda}$ and a fixed income vector $\boldsymbol{y}_{-i}^0$. Let $\kappa_{ij} \geq 0$ be the capacity of link ij in the social collateral model, with $\kappa_{ij} = 0$ if i and j are not connected. The problem of the planner we consider is to maximize $\sum_i \lambda_i \mathbb{E} U_i(y_i)$ subject to $y_i = y_i^0 + \sum_j t_{ji} - \sum_j t_{ij}$, and incentive compatibility constraints $t_{ij} \leq \kappa_{ij}$. Because of additive separability, the problem can be solved ex post for each income realization.

By incentive compatibility in the social collateral model, the highest feasible consumption for i is given by $y_i = y_i^0 + \sum_l \kappa_{li}$, and the lowest feasible consumption level for any agent j is given by $y_j = y_j^0 - \sum_l \kappa_{jl}$. Define the weighted marginal utility function of any agent j by $F_j(x) = \lambda_j U_j'(x)$, which, by the CARA assumption, is strictly decreasing. We define

$$Y_L = F_i^{-1} \left(\max_{j: \kappa_{ji} > 0} F_j(y_j^0 - \sum_l \kappa_{jl}) \right) - \sum_l \kappa_{li}.$$

Then for $y_i^0 \leq Y_L$, we show that any constrained efficient risk sharing agreement for weights $\boldsymbol{\lambda}$ must satisfy $y_i = y_i^0 + \sum_l \kappa_{li}$. Indeed, suppose $\boldsymbol{y}$ is the consumption vector associated with income realization $y_i^0 \leq Y_L$ for some constrained efficient risk-sharing agreement and $y_i < y_i^0 + \sum_l \kappa_{li}$. For this to be true, there must be some j connected to i such that either (i) $t_{ji} < \kappa_{ji}$, or (ii) $t_{ij} > 0$. But then, by construction, we have

$$F_j(y_j) < F_i(y_i),$$

which implies that increasing j's transfer to i in case (i), or decreasing t_{ij} in case (ii), would increase the planner's utility. Since both operations are feasible, this contradicts the constrained optimality of $\boldsymbol{y}$. The proof is similar for high income realizations for i.

Proof of Theorem 1.

Lemma A1 *Fix a transfer graph* $\mathbf{G}$. *For any* i, j, k, *we have:* $\bar{c}_{ji} = -\bar{c}_{ij}$, $\bar{c}_{ij} + \bar{c}_{jk} = \bar{c}_{ik}$ *and* $\ln(\lambda_i) - \ln(\lambda_j) = \bar{c}_{ij}$. *Further,* $\sum_i \ln(\lambda_i) = 0$.

Proof: (1) The path leading from j to i reverses all directions from the path leading from i to j, leading to the first property. (2) Suppose that j lies on the path connecting i to k. By definition, $\bar{c}_{ik} = \bar{c}_{ij} + \bar{c}_{jk}$. If k lies on the path connecting i to j, we then have $\bar{c}_{ij} = \bar{c}_{ik} + \bar{c}_{kj} = \bar{c}_{ik} - \bar{c}_{jk}$. Next, suppose that l is the last agent lying both on the path from i to k and on the path from i to j. Then, $\bar{c}_{ik} = \bar{c}_{il} + \bar{c}_{lk}$ and $\bar{c}_{ij} = \bar{c}_{il} + \bar{c}_{lj}$. Moreover, the path from k to j is formed of the path from k to l and of the path from l to j. Therefore, $\bar{c}_{kj} = \bar{c}_{kl} + \bar{c}_{lj}$. This yields: $\bar{c}_{ik} + \bar{c}_{kj} = \bar{c}_{il} + \bar{c}_{lk} + \bar{c}_{kl} + \bar{c}_{lj} = \bar{c}_{il} + \bar{c}_{lj} = \bar{c}_{ij}$. (3) Applying these two properties, we obtain:
$\ln(\lambda_i) - \ln(\lambda_j) = \frac{1}{n_C}\sum_{k\in C}(\bar{c}_{ik} - \bar{c}_{jk}) = \frac{1}{n_C}\sum_{k\in C}(\bar{c}_{ik} + \bar{c}_{kj}) = \frac{1}{n_C}\sum_{k\in C}\bar{c}_{ij} = \bar{c}_{ij}$
(4). Finally, note that $\sum_i \ln(\lambda_i) = \frac{1}{n_C}\sum_{i,j}\bar{c}_{ij} = \frac{1}{n_C}\sum_{i<j}(\bar{c}_{ij} + \bar{c}_{ji}) = 0$. QED.

Lemma A2 *Consider an income realization* $\boldsymbol{y}^0$, *equilibrium transfers* $\mathbf{T}$ *with transfer graph* $\mathbf{G}$. *Let* C *be a weak component of* $\mathbf{G}$. *Then, equilibrium consumption profile* $\boldsymbol{y}_C$ *on* C *solves the planner's program:* $\max_{\tilde{\boldsymbol{y}}_C}\sum_i \lambda_i u_i(\tilde{y}_i)$ *under the constraint* $\sum_{i\in C}\tilde{y}_i = \sum_{i\in C} y_i^0$ *and with* λ_i *such that* $\ln(\lambda_i) = \frac{1}{n_C}\sum_{j\in C}\bar{c}_{ij}$.

Proof: Consider i and j in C, connected through the path $i_1 = i$, i_2,..., $i_l = j$. If $g_{i_s i_{s+1}} = 1$, then equilibrium conditions imply that $\ln(u'_{i_s}(y_{i_s})) - \ln(u'_{i_{s+1}}(y_{i_{s+1}})) = -c_{i_s i_{s+1}}$. If $g_{i_{s+1} i_s} = 1$, then $\ln(u'_{i_s}(y_{i_s})) - \ln(u'_{i_{s+1}}(y_{i_{s+1}})) = c_{i_{s+1} i_s}$. Summing over all agents in the path yields

$$\ln(u'_i(y_i)) - \ln(u'_j(y_j)) = -\bar{c}_{ij} = \ln(\lambda_j) - \ln(\lambda_i)$$

by Lemma A1. These correspond to the first-order conditions of the planner's program. In addition, no money flows from C to $N - C$ or from $N - C$ to C. Therefore, $\sum_{i\in C} y_i^0 = \sum_{i\in C} y_i$ and aggregate income is preserved within C. QED.

Suppose that for any income realization, there is a Nash equilibrium with transfer graph $\mathbf{G}$. Then, the λ_i's do not depend on the income realization and the first part of Theorem 1 follows directly from Lemma A2.

For the second part, consider an altruism network $\boldsymbol{\alpha}$ satisfying the following property. Consider an undirected cycle, that is, a binary graph $\mathbf{U}$ connecting l agents i_1,..., $i_l = i_1$ such that either $u_{i_s i_{s+1}} = 1$ or $u_{i_{s+1} i_s} = 1$ and $u_{ij} = 0$ if i and j are not two consecutive agents in the set . Then, $\sum_{s:u_{i_s i_{s+1}}=1} c_{i_s i_{s+1}} - \sum_{s:u_{i_{s+1} i_s}=1} c_{i_{s+1} i_s} \neq 0$. In Bourlès, Bramoullé & Perez-Richet (2017), we showed that such networks are generic and that they always have a unique Nash equlibrium. Given a binary directed forest $\mathbf{G}$, define $Y_0(\mathbf{G}) = \{\boldsymbol{y}^0 \in Y_0 :$ the transfer graph of the Nash equilibrium is $\mathbf{G}\}$ the set of income realizations leading to $\mathbf{G}$ and $\mathring{Y}_0(\mathbf{G})$ its interior. Observe that the non-empty sets $Y_0(\mathbf{G})$ define a finite partition of Y_0. Define $\boldsymbol{\lambda}(\mathbf{G})$ the profile of Pareto weights as defined in the first part of Theorem 1. This mapping satisfies the following useful property.

Lemma A3 *Consider two binary directed trees* $\mathbf{G}$ *and* $\mathbf{H}$. *Then,* $\boldsymbol{\lambda}(\mathbf{G}) = \boldsymbol{\lambda}(\mathbf{H}) \Rightarrow \mathbf{G} = \mathbf{H}$.

Proof: Let $\boldsymbol{\lambda} = \boldsymbol{\lambda}(\mathbf{G}) = \boldsymbol{\lambda}(\mathbf{H})$ and suppose that $\mathbf{G} \neq \mathbf{H}$. There exists i, j such that $g_{ij} = 1$ and $h_{ij} = 0$. Since $g_{ij} = 1$, $\ln(\lambda_i) - \ln(\lambda_j) = c_{ij}$. Since $\boldsymbol{\lambda} = \boldsymbol{\lambda}(\mathbf{H})$, $\ln(\lambda_i) - \ln(\lambda_j) = \bar{c}_{ij} = \sum_{s:h_{i_s i_{s+1}}=1} c_{i_s i_{s+1}} - \sum_{s:h_{i_{s+1} i_s}=1} c_{i_{s+1} i_s}$ for an undirected path connecting i to j. The set $i, i_2, ..., i_l = j, i$ then defines an undirected cycle satisfying $\sum_{s:u_{i_s i_{s+1}}=1} c_{i_s i_{s+1}} - \sum_{s:u_{i_{s+1} i_s}=1} c_{i_{s+1} i_s} = 0$, which is impossible given our assumptions on $\boldsymbol{\alpha}$. QED.

Suppose first that there is only one community in the partition. In other words, altruistic transfers generate efficient insurance for Pareto weights $\boldsymbol{\mu}$. This implies that there exist functions f_i such that $\forall \boldsymbol{y}^0 \in Y_0, y_i = f_i(\sum_j y_j^0)$. Let $\mathbf{G}$ be any graph such that $\mathring{Y}_0(\mathbf{G}) \neq \varnothing$. Such a graph exists by the assumption that $\mathring{Y}_0 \neq \varnothing$. Suppose that $\mathbf{G}$ is disconnected. Then by the first part of Theorem 1, there exist functions g_i such that $\forall \boldsymbol{y}^0 \in \mathring{Y}_0(\mathbf{G})$, $y_i = g_i(\sum_{j \in C} y_j^0) = f_i(\sum_{j \in C} y_j^0 + \sum_{j \in N-C} y_j^0)$. This implies that $\forall \boldsymbol{y}^0 \in \mathring{Y}_0(\mathbf{G}), \sum_{j \in N-C} y_j^0 = L$ which contradicts the fact that $\mathring{Y}_0(\mathbf{G})$ is a non-empty open set.

Therefore, $\mathbf{G}$ is connected and hence is a tree. By the first part of Theorem 1, there is efficient risk sharing on $\mathring{Y}_0(\mathbf{G})$ for Pareto weights $\boldsymbol{\lambda}(\mathbf{G})$. $\forall i, j, u_i'(y_i)/u_j'(y_j) = \lambda_j/\lambda_i = \mu_j/\mu_i$. This implies that there exists $t > 0$ such that $\boldsymbol{\mu} = t\boldsymbol{\lambda}(\mathbf{G})$. Next consider another graph $\mathbf{H}$ for which $\mathring{Y}_0(\mathbf{H}) \neq \varnothing$. By the same reasoning, there exists $t' > 0$ such that $\boldsymbol{\mu} = t'\boldsymbol{\lambda}(\mathbf{H})$. Since $\boldsymbol{\lambda}(\mathbf{G})$ and $\boldsymbol{\lambda}(\mathbf{H})$ satisfy the same normalization $\sum_j \ln(\lambda_j) = 0$, then $\boldsymbol{\lambda}(\mathbf{G}) = \boldsymbol{\lambda}(\mathbf{H})$. By Lemma A3, $\mathbf{G} = \mathbf{H}$. Therefore, $\mathring{Y}_0 = \mathring{Y}_0(\mathbf{G})$.

Finally, suppose that the partition is composed of several communities. Apply, first, the previous reasoning to each community C considered separately. There exists a tree graph $\mathbf{G}_C$ connecting agents in C and such that $\boldsymbol{\mu}_C = t_C \boldsymbol{\lambda}(\mathbf{G}_C)$ for $t_C > 0$ and $\boldsymbol{\mu}_C$ Pareto weights within C and $\mathbf{G}_C$ describes the pattern of giving relationships within C. Next, let us show that for any income realization in the support's interior, an agent in one community cannot give to an agent in another. Constrained efficiency implies income conservation within communities: $\forall C, \sum_{i \in C} y_i = \sum_{i \in C} y_i^0$. Suppose that for some $\boldsymbol{y}^0 \in Y_0$, there are some intercommunity transfers. The graph connecting communities is also a forest. Therefore, there exists a community connected to other communities through a single link. Formally, there exists $C \neq C'$ such that $i \in C$, $j \in C'$ and $t_{ij} > 0$ or $t_{ji} > 0$ and where there is no other giving link connecting C and $N - C$. If $t_{ij} > 0$, this implies $\sum_{i \in C} y_i = \sum_{i \in C} y_i^0 - t_{ij}$. If $t_{ji} > 0$, this implies $\sum_{i \in C} y_i = \sum_{i \in C} y_i^0 + t_{ij}$. In either case, $\sum_{i \in C} y_i \neq \sum_{i \in C} y_i^0$, which contradicts the original assumption. QED.

Proof of Proposition 5. If α_{ij} increases, c_{ij} decreases. Then, $\bar{c}_{kl}$ decreases if the link ij lies on the path connecting k to l. By contrast, $\bar{c}_{kl}$ increases if the link ji lies on the path connecting k to l. Agents in C_i are connected through agents in C_j through the link ij, and this link does not appear on the path connecting agents in C_i. This implies that $\sum_{l \in C} \bar{c}_{kl}$ decreases if $k \in C_i$. Similarly, $\sum_{l \in C} \bar{c}_{kl}$ increases if $k \in C_j$. QED.

A new connection can increase consumption variance under income correlation. Consider agents 1, 2 and 3 with incomes $(12, 0, 0)$ with probability $1/2$ and $(0, 12, 12)$ with probability $1/2$. Note that this satisfies the symmetry assumption of Proposition 1. Agents have common CARA utilities with $-\ln(\alpha)/A = 2$. In the original network, 1 and 2 are connected and 3 is isolated. Consumption is $(7,5,0)$ with proba $1/2$ and $(5,7,12)$ with proba $1/2$. Next, connect 2 and 3. Consumption becomes $(6,4,2)$ with proba $1/2$ and $(6,8,10)$ with proba $1/2$. Agent 2 faces a more risky consumption profile. Here, the income streams of agent 2 and 3 are perfectly positively correlated. Agent 2's consumption becomes lower when poor and higher when rich, due to this new connection.

Variance computations on p.25. With 3 agents, there are 8 states of the world. Consider, first, the network where 1 and 2 are connected and 3 is isolated, see the example in Section 2.1. Since $c < 2\sigma$, the variance of y_1 and y_2 drops from σ^2 to $\frac{1}{2}\sigma^2 + \frac{1}{4}c^2$. Next, connect agents 2 and 3 to form a line. We assume that altruism is high enough to induce transfer paths of length 2 in situations where a single peripheral agent has a positive or a negative shock. This is satisfied iff $c < \frac{2}{3}\sigma$. Computing transfers and consumption for each state of the world, we find, $Var(y_1) = Var(y_3) = \frac{1}{3}\sigma^2 + \frac{1}{18}\sigma c + \frac{19}{36}c^2$ and $Var(y_2) = \frac{1}{3}\sigma^2 - \frac{1}{9}\sigma c + \frac{1}{9}c^2$. All variances drop. Finally, connect agents 1 and 3 to form the triangle. Consumption variance for any agent is now equal to $\frac{1}{3}\sigma^2 + \frac{1}{6}c^2$. $Var(y_2)$ increases while $Var(y_1) = Var(y_3)$ decreases. QED.

Proof of Proposition 6. Our proof makes use of the following classical properties of the covariance operator, see e.g. Gollier (2001). First, if f and g are non-decreasing functions and $\tilde{X}$ is some random variable, then, $cov(f(\tilde{X}), g(\tilde{X})) \geq 0$. Second, the law of total covariance states that if $\tilde{X}, \tilde{Y}, \tilde{Z}$ are three random variables, then $cov(\tilde{X}, \tilde{Y}) = \mathbb{E}(cov(\tilde{X}, \tilde{Y}|Z)) + cov(\mathbb{E}(\tilde{X}|Z), \mathbb{E}(\tilde{Y}, Z))$.

Given set of agents S, denote by $\boldsymbol{y}^0_{-S}$ the vector of incomes of agents not in S. Apply the law of total covariance to variables $\tilde{y}_i, \tilde{y}_j$ and $\tilde{\boldsymbol{y}}^0_{-1}$. This yields $cov(\tilde{y}_i, \tilde{y}_j) = \mathbb{E}(cov(\tilde{y}_i, \tilde{y}_j|\boldsymbol{y}^0_{-1})) + cov(\mathbb{E}(\tilde{y}_i|\boldsymbol{y}^0_{-1}), \mathbb{E}(\tilde{y}_j|\boldsymbol{y}^0_{-1}))$. Note that conditional on $\boldsymbol{y}^0_{-1}$, y_i and y_j are deterministic, non-decreasing functions of y^0_1 by Theorem 3 in Bourlès, Bramoullé & Perez-Richet (2017). By the property of the covariance of monotone functions, this implies that $\forall \boldsymbol{y}^0_{-1}$, $cov(\tilde{y}_i, \tilde{y}_j|\boldsymbol{y}^0_{-1}) \geq 0$ and hence $\mathbb{E}(cov(\tilde{y}_i, \tilde{y}_j|\boldsymbol{y}^0_{-1})) \geq 0$. Next, let f_1 denote the pdf of $\tilde{y}^0_1$. By independence,

$$\mathbb{E}(\tilde{y}_i|\boldsymbol{y}^0_{-1}) = \int y_i(y^0_1, \boldsymbol{y}^0_{-1}) f_1(y^0_1) dy^0_1$$

Since $y_i(y^0_1, \boldsymbol{y}^0_{-1})$ is non-decreasing in y^0_2, this implies that $\mathbb{E}(\tilde{y}_i|\boldsymbol{y}^0_{-1})$ is also non-decreasing in y^0_2. We can therefore repeat the argument: $cov(\mathbb{E}(\tilde{y}_i|\boldsymbol{y}^0_{-1}), \mathbb{E}(\tilde{y}_j|\boldsymbol{y}^0_{-1})) = \mathbb{E}(cov(\tilde{y}_i, \tilde{y}_j|\boldsymbol{y}^0_{-1,2})) + cov(\mathbb{E}(\tilde{y}_i|\boldsymbol{y}^0_{-1,2}), \mathbb{E}(\tilde{y}_j|\boldsymbol{y}^0_{-1,2}))$ where $\mathbb{E}(cov(\tilde{y}_i, \tilde{y}_j|\boldsymbol{y}^0_{-1,2})) \geq 0$ by monotonicity. Dimensionality is reduced at each step, and all terms are non-negative. QED.

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