

# ORGANIZATION AND REGULATION OF FINANCIAL SYSTEMS

S8 DMC

*Renaud Bourlès*

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## OUTLINE

### ▷ INTRODUCTION

- ▶ Lessons from the crisis
- ▶ What is a bank and what do banks do?

### ▷ MODELS FOR BANKING REGULATION

- ▶ Deposit insurance
- ▶ Lender of last resort: a simple model

### ▷ CAPITAL RESERVES: THE CASE OF INSURANCE

- ▶ Optimal choice of capital reserve
- ▶ Failure risk and insurance demand

## LESSONS FROM THE (LAST) CRISIS

see *Tirole*, in "*Balancing the Banks*", or

*Beneplanc and Rochet: "Risk management in turbulent times"*

### ▷ **LAST** crisis

- ▶ since 1970: 112 banking crises, affecting 93 countries
- ▶ 51 international crises (affecting several countries)

### ▷ Financial **MADNESS?**

- ▶ ECON 101: all economic agents  
(incl. managers and employees in financial industries)
- ▶ react to the information and incentives

▷ Bad incentives + bad information  $\Rightarrow$  **BAD BEHAVIOR**

## WHAT HAPPENED?

- ▷ **ORIGIN** : home loans market
- ▷ then:
  - ▶ sale of assets at **FIRE-SALE PRICES**
  - ▶ unprecedented **AVERSION TO RISK**
  - ▶ **FREEZING OF INTERBANK** and bond market
- ▷ "government" **REACTION**: bail-out ("renflouement") of some of the largest banks and a major insurance company

## AN EXAMPLE: AIG

- ▷ Beginning of 2007
    - ▶ \$ 1 trillion of assets
    - ▶ \$ 110 billion revenue
    - ▶ 74 million customers
  - ▷ September 2008: emergency government assistance
    - ▶ 2-year emergency loan of \$ 85 billion
    - ▶ gvt hold 79.9% of shares
- ⇒ 50% of U.S. GDP has been **GUARANTEED**, **LENT** or spent by the Fed, the US Treasury and other federal agencies

## THE ROLE OF SUBPRIME MORTGAGES

- ▷ **Sub**prime mortgages ("prêt hypothécaires"): loans w/ difficulties in maintaining repayment schedule
  - ▶ higher interest rate
  - ▶ less favorable terms (collateral)

to **COMPENSATE** for high risk

- ▷ losses on the US subprime market **SMALL** relative to previous figures (\$1,000 billion, 4% of NYSE capitalization)

= detonator for a sequence of incentives and market **FAILURES** (asym. info. betw/ contracting parties) exacerbated by bad news

## OTHER ISSUES

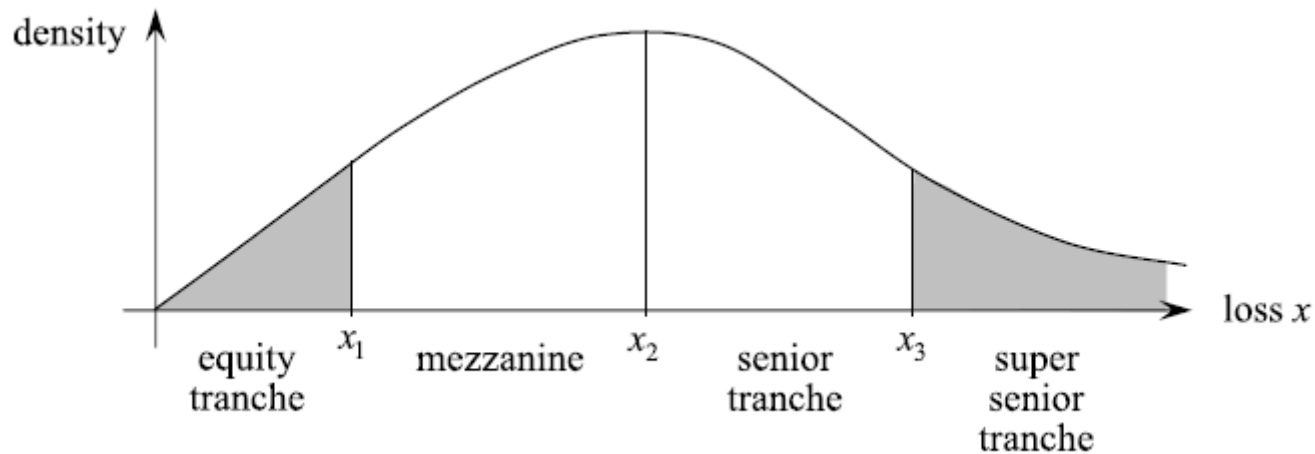
- ▷ bad **REGULATION** → incentives to take risk
- ▷ **POLITICAL** resolution to favor real estate  
(to promote acquisition of homes by households)
- ▷ **MONETARY POLICY**: short term interest rate low
- ▷ excessive **LIQUIDITY**
  - ▶ international savings → US ⇒ excess liquidity
  - ⇒ **SECURIZATION** ("titrisation") to answer the demand

## SECURIZATION

### ▷ Aim

- ▶ to refinance the lender → can finance other activities
- ▶ to fulfill the demand for securities
- ▶ to diversify and spread risk

### ▷ example: **TRANCHING**



[equity ("fond propre") tranche generally retained by the bank]



## SECURIZATION: CDO

### ▷ Collateralized Debt Obligation

- ▶ the bank issues bonds against investment
- ▶ **PRIORITIZED** by different tranches
- ▶ ex: 3 loans of nominal 1, each w/ proba 10% of default and 0 recovery in case of default
- ▶  $\mathbb{P}(i \text{ defaults}) = \binom{i}{n} p^i (1 - p)^{n-i}$   
 $\mathbb{P}(1 \text{ d}) = 24.3\%$ ,  $\mathbb{P}(2 \text{ d}) = 2.7\%$ ,  $\mathbb{P}(3 \text{ d}) = 0.1\%$
- ▶ equity tranche: loss up to 1  
mezzanine tranche: loss between  $x_1 = 1$  and  $x_2 = 2$   
senior: losses above  $x_2 = 2$

## SECURIZATION: CDS

### ▷ Credit Default Swap

- ▶ contract between two parties
- ▶ the **PROTECTION** buyers pays a period premium
- ▶ to the protection seller who, in exchange,
- ▶ commit to pay a fixed sum if a credit instrument (a bond or a loan) **DEFAULT**

### ▷ different for insurance

- ▶ the buyer **DOESN'T NECESSARILY OWN** the credit instrument
- ▶ the seller is **NOT A REGULATED** entity

## SECURIZATION: ISSUES

- ▷ shift the **RESPONSIBILITY** away from the lender  
⇒ less incentive to **CONTROL**
- ▷ asymmetry of **INFORMATION**
- ▷ laxity of credit-rating **AGENCIES**
- ▷ excessive maturity transformation

## THE NORTHERN ROCK EXAMPLE

- ▷ **STRATEGY:** invest in (apparently) safe tranches of Residential Mortgage Backed Securities (RMBS)
  - ▷ financed by short term deposit
  - ▷ **PROBLEM:** rumors (risk on RMBS)  $\Rightarrow$  panic  $\Rightarrow$  bank run
- $\Rightarrow$  nationalization: injection of £23 billion
- ▷ lack of liquidity also led to default of **LEHMAN BROTHERS** (biggest default in the US history: \$ 613 bn of debt)

## HOW TO REGULATE?

- ▷ Basel accords: **REQUIREMENT** regarding the minimal level of **CAPITAL** or equity ("fonds propres")
- ▷ Basel I: requires **8% OF BANK CREDIT RISK**
- ▷ Problems
  - ▶ **OTHER RISKS?** Liquidity risks? Off balance-sheet?
  - ▶ Risk **MEASURE?**
  - ▶ **INFORMATION**
  - ▶ **INCENTIVES.** Ex: managerial incentives (stock options). The CEO of Lehman Brothers earned \$ 250 million between 2004 and 2007
  - ▶ **SYSTEMIC** institutions: Too Big To Fail

## BASEL II

▷ published in 2004, "implemented" in 2008

Pillar I <b>MINIMAL CAPITAL REQUIREMENT</b>	Pillar II <b>SUPERVISORY REVIEW PROCESS</b>	Pillar III <b>DISCLOSURE REQUIREMENT</b>
<ul style="list-style-type: none"> <li>▷ Credit Risk</li> <li>▷ Market Risk</li> <li>▷ Operational Risk</li> </ul>	<ul style="list-style-type: none"> <li>▷ Regulatory framework               <ul style="list-style-type: none"> <li>▶ Internal cap. adequacy</li> <li>▶ Risk management</li> </ul> </li> <li>▷ Supervisory framework               <ul style="list-style-type: none"> <li>▶ Evaluation of internal systems</li> <li>▶ Assessment of risk profile</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>▷ Disclosure on capital, risk exposures, risk assessment process capital adequacy</li> <li>▷ Comparability</li> </ul>

▷ **BASEL III** published in 2010, not yet implemented

▷ tries to also account for **LIQUIDITY RISK**

▷ and **SIFIs** (Systemically important financial institutions)

## WHAT IS A BANK AND WHAT DO BANKS DO? (1)

see *Freixas and Rochet: "Microeconomics of banking"*

▷ Banking operations **VARIED AND COMPLEX**

▷ But a **SIMPLE** operational def (used by regulators) is

"a bank is an institution whose current operations consist in granting loans and receiving deposits from the public"

▷ **CURRENT** important: most firms occasionally lend money to customers or borrow from suppliers.

▷ **BOTH LOANS AND DEPOSITS** important: combination of lending and borrowing typical of commercial banks. Finance a significant share of loans through deposits → fragility.

▷ **PUBLIC**: not armed ( $\neq$  professional investors) to assess safety financial institutions. Public good (access to safe and efficient payment system) provided by private institutions

## WHAT IS A BANK AND WHAT DO BANKS DO? (2)

- ▷ Protection of depositors + safety and efficiency of payment system → **PUBLIC INTERVENTION**
- ▷ Crucial role in **ALLOCATION OF CAPITAL**
  - ▶ efficient life-cycle allocation of household consumption
  - ▶ efficient allocation of capital to its most productive use
- ▷ before performed by banks alone; now fin. markets also
- ▷ **4 FUNCTIONS** performed by banks
  - ▶ Offering liquidity and payment services
  - ▶ Transforming assets
  - ▶ Managing risks
  - ▶ Processing information and monitoring borrowers



## LIQUIDITY AND PAYMENT SERVICES

- ▷ Without transaction costs (Arrow-Debreu): no need for money.
- ▷ **FRICTIONS** → more efficient to exchange goods for money.
- ▷ commodity money ("m. marchandise") → fiat money ("m. fiduciaire"): medium of exchange, intrinsically **USELESS**, guaranteed by some institution
- ▷ Role of **BANKS**
  - ▶ money change (exchange between different currencies issued by distinct institutions) ⇒ dvlp of trade
  - + management of deposits (less liquid, safer)
  - ▶ payment services: species inadequate for **LARGE** or at distance payments
  - banks played an important part in clearing positions

## TRANSFORMING ASSETS

Asset transformation can be seen from three viewpoints:

- ▷ convenience of **DENOMINATION** (size). Ex: small depositors facing large investors willing to borrow indivisible amounts.
- ▷ **QUALITY** transformation: better risk-return characteristics than direct investments (diversified portfolio, better info)
- ▷ **MATURITY** transformation: transforms short maturities (deposits) into long maturities (loans) → risk of illiquidity  
**SOLUTION**: interbank lending and derivative financial instruments (swaps, futures)

## MANAGING RISK

- ▷ Credit risk  $\Rightarrow$  use of **COLLATERAL**
- ▷ Liquidity risk  $\Rightarrow$  interest rate
- ▷ Off-Balance-sheet risk: **COMPETITION**  $\Rightarrow$  more sophisticated contracts
  - ▶ loan commitment, credit lines
  - ▶ guarantees and swaps (CDS)
  - ▶ hedging contracts ("opération de couverture")
- ▷ not real liability (or asset): **CONDITIONAL COMMITMENT**  
 $\Rightarrow$  need of careful **REGULATION**

## MONITORING AND INFORMATION PROCESSING

- ▷ Problems resulting from **IMPERFECT INFORMATION** on borrowers.
- ⇒ Banks invest in technologies that allow them
  - ▶ to **SCREEN** loan applicants and
  - ▶ to **MONITOR** their projects
- ▷ Long-term relationships: mitigates **MORAL HAZARD**

## A SIMPLE MODEL WITH MORAL HAZARD

- ▷ Firms seek to finance investment projects of a size 1
- ▷ Risk-free rate of interest normalized to zero.
- ▷ Firms have **CHOICE** between
  - ▶ a good technology:  $G$  with proba.  $\Pi_G$  (0 otherwise)
  - ▶ a bad technology:  $B$  with proba.  $\Pi_B$  (0 otherwise)
- ▷ Only  $G$  proj. have positive net (expected) present value:

$$\Pi_G G > 1 > \Pi_B B$$

but  $B > G$ , (which implies  $\Pi_G > \Pi_B$ )

- ▷ Success verifiable, not choice of techno. (nor return)
- can promise to repay  $R$  (nominal debt) only if success
- + no other source of cash → repayment zero if fails
- ▷ value of  $R$  determines choice of **TECHNOLOGY**

## IN THE ABSENCE OF MONITORING

▷ chooses G techno. iif gives higher expected profit:

$$\Pi_G(G - R) > \Pi_B(B - R)$$

▷ Since  $\Pi_G > \Pi_B$  this is equivalent to

$$R < R_C \equiv \frac{\Pi_G G - \Pi_B B}{\Pi_G - \Pi_B}$$

⇒ Proba  $\Pi$  of **REPAYMENT DEPENDS ON R**:

$$\Pi(R) = \begin{cases} \Pi_G & \text{if } R \leq R_C \\ \Pi_B & \text{if } R > R_C \end{cases}$$

▷ Competitive equilibrium  $\rightarrow \Pi(R) \cdot R = 1$

▷ as  $\Pi_B R < 1 \forall R < D$ , **ONLY POSSIBLE EQ.: G**

▷ works only if:  $\Pi_G R_C \geq 1$ , i.e.  $R_C$  high enough

↔ if **MORAL HAZARD NOT TOO IMPORTANT**

▷ otherwise: no trade (no credit market)

## INCLUDING MONITORING

▷ at cost  $C$ , **BANKS** can prevent from using bad techno

⇒ new equilibrium interest rate:  $\Pi_G R_m = 1 + C$

▷ bank lending appear at equilibrium if (as  $R_m < G$ ):

▶  $\boxed{\Pi_G G > 1 + C}$   $\leftrightarrow$  monitoring cost lower than the NPV

▶  $\boxed{\Pi_G R_C < 1}$   $\leftrightarrow$  direct lending (less expensive) not possible

▷ that is for intermediate values of  $\Pi_G$ :

$$\Pi_G \in \left[ \frac{1 + C}{G}, \frac{1}{R_C} \right]$$

## CONCLUSION

▷ Assuming the monitoring cost  $C$  small enough so that

$$\frac{1}{R_C} > \frac{1+C}{G}$$

▷ 3 possible regimes of the credit market at equilibrium:

▶ if  $\Pi_G > \frac{1}{R_C}$ : firms issue direct debt at rate  $R_1 = \frac{1}{\Pi_G}$

▶ if  $\Pi_G \in \left[\frac{1+C}{G}, \frac{1}{R_C}\right]$ : borrow from **BANKS** at rate  $R_2 = \frac{1+C}{\Pi_G}$

▶ if  $\Pi_G < \frac{1+C}{G}$ : credit market collapses (no trade eq.)



## POSSIBLE EXTENSIONS

- ▷ Dynamic model (2 dates) with **REPUTATION**
    - ▶ repayment at  $t = 1 \rightarrow$  possibility of (cheaper) direct loan at  $t = 2$
    - ▶  $R^{t=1} < R_C$  (reputation  $\downarrow$  moral hazard);  $R_U^{t=2} > R_C$
  - ▷ Use of **CAPITAL** (choice between capital and debt)
    - ▶ well capitalized  $\rightarrow$  direct loan
    - ▶ intermediate capitalization  $\rightarrow$  bank loan
    - ▶ under-capitalized  $\rightarrow$  no loan
- $\rightarrow$  substitutability between capital and monitoring

## RECALL: WHY TO REGULATE? (1)

- ▷ In general: **WELFARE** theorems
- ▷ Regulation iif **MARKET FAILURES**:  
externalities, asymmetric information
- ▷ Banks (or fin. intermediaries) solve some of these problems
- ▷ **BUT** create others:
  - ▶ liquidity risk: assets illiquid, liabilities liquid

Assets	Liabilities
	Deposits
Loans	
Reserves	Capital (bonds)

## RECALL: WHY TO REGULATE? (2)

- ▷ to **PROTECT CLIENTS** (small depositors)
  - ▶  $\neq$  other institutions: creditors = public
  - no monitoring power
  - ▶ creditor of other firms: **BANKS** (can monitor)
- + **CONFLICT OF INTERESTS** btw/ manager and depositors  
managers take too much risk (not their mean of payment)
- + **COST OF FAILURE**: contagion + **CONFIDENCE** on the system of payment
- ⇒ **DEPOSIT INSURANCE** + **LENDER OF LAST RESORT** + **CAPITAL RATIO** (+ Takeover ultimately)

## DEPOSIT INSURANCE

- ▷ to avoid bank panics and their social costs
- ▷ governments have established deposit insurance schemes:
  - ▷ banks pay a premium to a deposit insurance fund
- ▷ ex Federal Deposit Insurance Corporation in the U.S.
  - ▶ created in 1933
  - ▶ in reaction to hundreds of failure in the 20s and 30s
- ▷ mostly public schemes
- ▷ pros
  - ▶ systemic risk → private sector not "credible"
  - ▶ take-off decisions = public
- ▷ cons: lack of competition
  - ▶ less incentive to extract info and price accurately

## DEPOSIT INSURANCE: A MODEL

see *Freixas and Rochet: "Microeconomics of banking" (section 9.3)*

▷ 2 dates:  $t = 0$  and  $t = 1$

▷ at  $t = 0$  the bank:

▶ issues equity  $E$

▶ receives deposits  $D$

▶ loans  $L$

▶ pays deposit **INSURANCE PREMIUMS**  $P$

Assets	Liabilities
Loans $L$	Deposits $D$
Insurance Premiums $P$	Equity $E$

- ▷ normalize the risk-free rate to 0
- ▷ at  $t = 1$  the bank is liquidated
- ▷ depositors **COMPENSATED** if bank's assets insufficient

Assets	Liabilities
Loan repayments $L_1$	Deposits $D$
Insurance payments $S$	Liquidation value $V$

- ▷ from  $t = 0$ :  $V$ ,  $S$  and  $L_1$  are stochastic:  $\tilde{V}$ ,  $\tilde{S}$  and  $\tilde{L}_1$
- ▷ with  $\tilde{V} = \tilde{L}_1 - D + \tilde{S}$
- ▷ insurance pays difference betw/ deposits (to "pay back") and loan repayments:

$$\tilde{S} = \max(0, D - \tilde{L}_1)$$

- ▷ moreover from  $t = 0$ :  $D = L + P - E$

▷ therefore:

$$\tilde{V} = E + (\tilde{L}_1 - L) + [\max(0, D - \tilde{L}_1) - P]$$

▷ shareholders' value of the bank = its initial value + the increase in the value of loans + net subsidy (<0 or >0) received from deposit insurance.

▷ if **FOR EXAMPLE**

$$\tilde{L}_1 = \begin{cases} X & \text{with prob. } \theta \\ 0 & \text{with prob. } 1 - \theta \end{cases}$$

▷ the **EXPECTED GAIN** for the bank's shareholders is

$$\begin{aligned} \mathbb{E}(\Pi) &\equiv \mathbb{E}(\tilde{V}) - E \\ &= \underbrace{(\theta X - L)}_{\text{net present value of loans}} + \underbrace{((1 - \theta)D - P)}_{\text{net subsidy from insurance}} \end{aligned}$$

$$\mathbb{E}(\Pi) = (\theta X - L) + ((1 - \theta)D - P)$$

▷ **PROBLEM:** create moral hazard

▶ Suppose  $P$  fixed, and

▶ banks choose characteristics  $(\theta, X)$  of projects

▶ Then, within projects with same NPV:  $\theta X - L = \text{cst}$

▶ they choose those with lowest  $\theta$  (i.e. **HIGHEST RISK**)

▷ **WHY?**

▶  $P/D$  (premium rate) does not depend **RISK TAKEN**

▶ as it was the case in the United States until 1991

▶ then new system with **RISK-RELATED** premiums



# LENDER OF LAST RESORT: A SOLUTION TO COORDINATION FAILURE

see *Rochet and Vives, JEEA 2004*

## MOTIVATION

- ▷ Role of government (or IMF):
- ▷ lend to banks "**ILLIQUID BUT SOLVENT**"!
- ▷ redundant w/ interbank market?
  - ▶ Yes! If the market works well
  - ▶ i.e. without asymmetric information
  - ▶ if it can recognize solvent banks

## LENDER OF LAST RESORT: THE MODEL

3 dates:  $\tau = 0, 1, 2$

▷ at  $\tau = 0$

- ▶ bank possesses own funds  $E$
- ▶ collects uninsured deposits  $D_0$  normalized to 1  
give  $D > 1$  when withdrawn (independ. of the date)
- ▶ used to finance investment  $I$  in risky assets (loans)
- ▶ the rest is held in cash reserves  $K$

▷ under normal circumstances:  $I \rightarrow \tilde{R}.I$  at  $\tau = 2$

deposits are reimbursed and shareholders get the difference

▷ BUT **ANTICIPATED WITHDRAWALS** (at  $\tau = 1$ ) can occur  
depending on the signal received by depositors on  $\tilde{R}$

▷ if proportion  $x > K$ : bank has to **SELL** part of its assets

## ASSUMPTIONS

- ▷ Withdrawal decision taken by **FUND MANAGERS**
  - ▶ in general they prefer not to do so
  - ▶ BUT are penalized by the investors if the **BANK FAILS**
- ▷ consistent with observations
  - ▶ majority of deposits held by collective investment funds
  - ▶ remuneration of fund managers based on size not return
- ▷ Model: remun. based on whether take the "right decision"
  - ▶ if withdraw and not fail  $\rightarrow -C$
  - ▶ if withdraw and fail  $\rightarrow B$
- ▷ noting  $P$  the probability that bank fails: withdraw if

$$PB - (1 - P)C > 0 \Leftrightarrow P > \gamma \equiv \frac{C}{B + C}$$

## SIGNALS AND FAILURE

At  $\tau = 1$

▷ manager  $i$  **PRIVATELY** observes a signal  $s_i = R + \varepsilon_i$   
with  $\varepsilon_i$  i.i.d. and indep. from  $R$

⇒  $x\%$  of the managers decide to withdraw

▷ if  $x > K/D$  the bank has to **SELL** a volume  $y$  of its asset  
(repurchase agreement  $\sim$  collateralized loan)

▶ if  $y > I$ : the bank **FAILS AT  $\tau = 1$**

▶ if  $R(I - y) < (1 - x)D$ : the bank **FAILS AT  $\tau = 2$**

## INTERBANK MARKET

- ▷ in case of **LIQUIDITY SHORTAGE** at  $\tau = 1$
- ▶ sell asset on repurchase agreement (or repo) market
  - ▶ informationally **EFFICIENT**: resale price depend on  $R$
  - ▶ BUT cost ( $\lambda$ ) of **FIRE-SALE** (or liquidity premium) the bank only gets a fraction  $\frac{1}{1+\lambda}$  of its asset value
- $$\Rightarrow y \quad / \quad \frac{Ry}{1+\lambda} = [xD - K]_+$$
- $$\Leftrightarrow y = (1 + \lambda) \frac{[xD - K]_+}{R}$$
- ▷  $\lambda$  is key to this analysis
- ▷ reflects e.g. moral hazard: 2 reasons for selling asset
- ▶ needs liquidity or wants to get rid of bad loans (value 0)
  - ▶  $\frac{1}{1+\lambda}$  is then the proba of the former

## AIM OF THE MODEL

- ▷ want to show that interbank market **DOES NOT SUFFICE**
- ▷ to prevent **EARLY CLOSURE** of the bank
- ▷ and so that we need a **LENDER OF LAST RESORT**
  
- ▷ if  $R$  small (close to insolvency) or  $\lambda$  large (liquidity shortage)
- ▷ even with interbank market: early closure at  $\tau = 1$
  
- ▷ Now: early closure  $\rightarrow$  physical **LIQUIDATION** of assets
- $\Rightarrow$  cost of liquidation ( $\neq \lambda$ )
  
- ▷ model: if a bank closes at  $\tau = 1$ , liquidation value  $\nu R$  with  
 $\nu \ll \frac{1}{1+\lambda}$

## BANK RUNS AND SOLVENCY (1)

▷ if  $xD \leq K$ : no sale of assets at  $\tau = 1$

⇒ failure at  $\tau = 2$  iif  $RI + K < D \Leftrightarrow R < \frac{D-K}{I} \equiv R_S$

▷ if  $K < xD \leq K + \frac{RI}{1+\lambda}$ : partial sale of assets at  $\tau = 1$

⇒ failure at  $\tau = 2$  iif

$$RI - (1+\lambda)(xD - K) < (1-x)D \Leftrightarrow R < R_S + \lambda \frac{xD - K}{I} \equiv R_F(x)$$

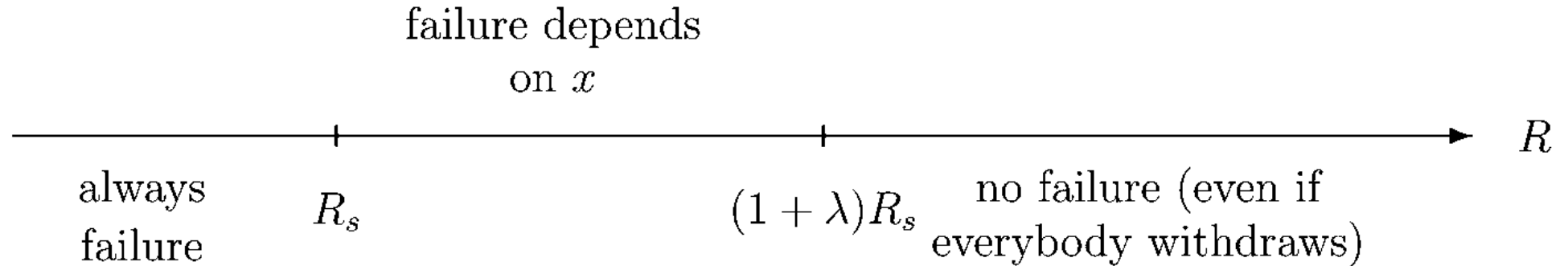
→ Because of  $\lambda$ , **SOLVENT** banks ( $R > R_S$ ) can fail

if  $R > (1 + \lambda)R_S$ , never fails (even  $x = 1$ ): super solvent

▷ if  $xD > K + \frac{RI}{1+\lambda}$ : failure at  $\tau = 1$

$$\Leftrightarrow R < (1 + \lambda) \frac{xD - K}{I} \equiv R_{EC}(x)$$

## BANK RUNS AND SOLVENCY (2)



▷ using the **LIQUIDITY RATIO**:  $k = K/D$ , we have:

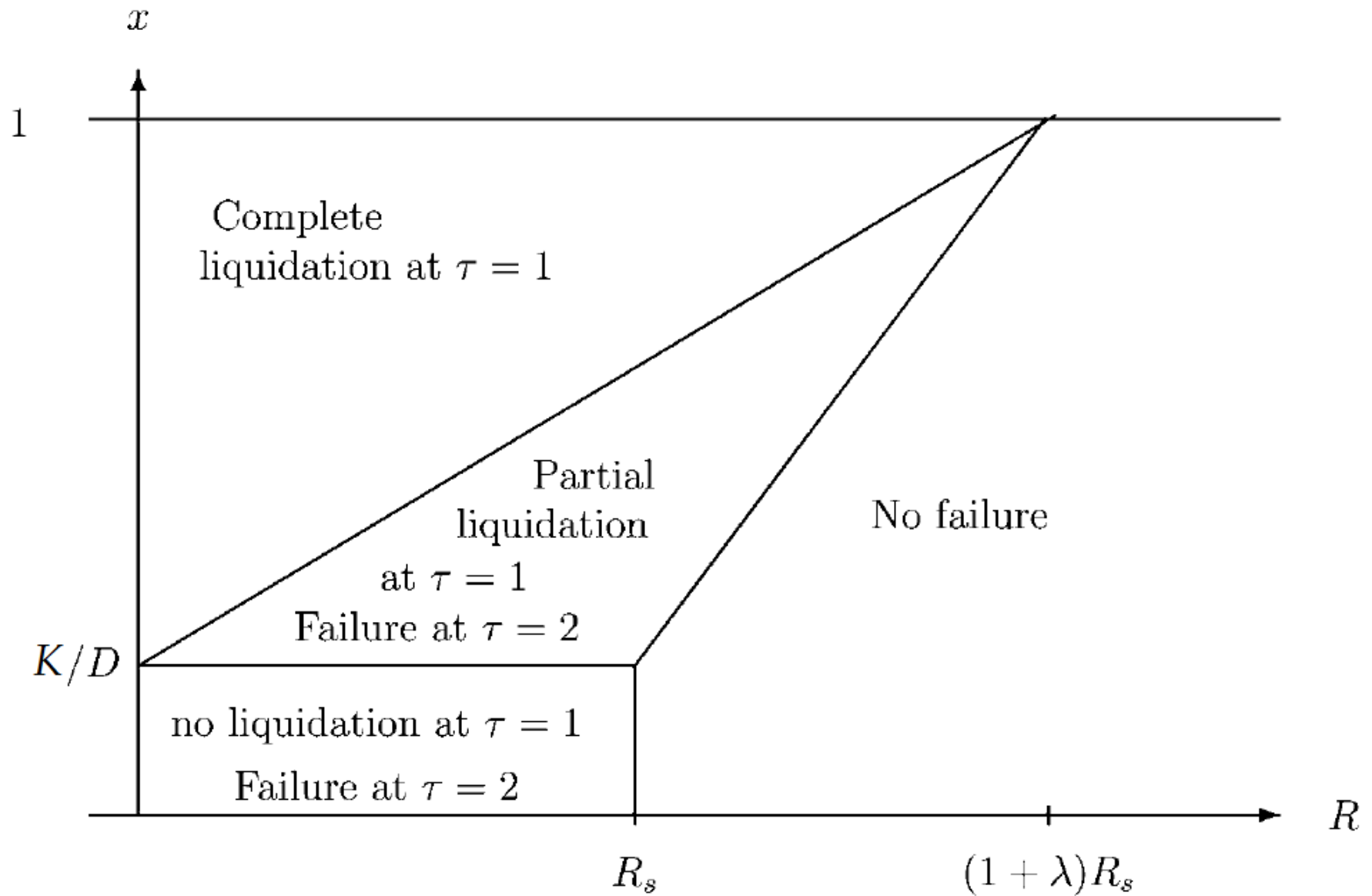
$$R_S = \frac{1-k}{I}D, \quad R_F(x) = R_S \left( 1 + \lambda \frac{[x-k]_+}{1-k} \right), \quad R_{EC}(x) = R_S (1 + \lambda) \frac{[x-k]_+}{1-k}$$

▷  $x \leq 1 \Rightarrow R_F(x) > R_{EC}(x)$

▷ **EARLY CLOSURE IMPLIES FAILURE**  
(the converse is not true)



## BANK RUNS AND SOLVENCY (3)

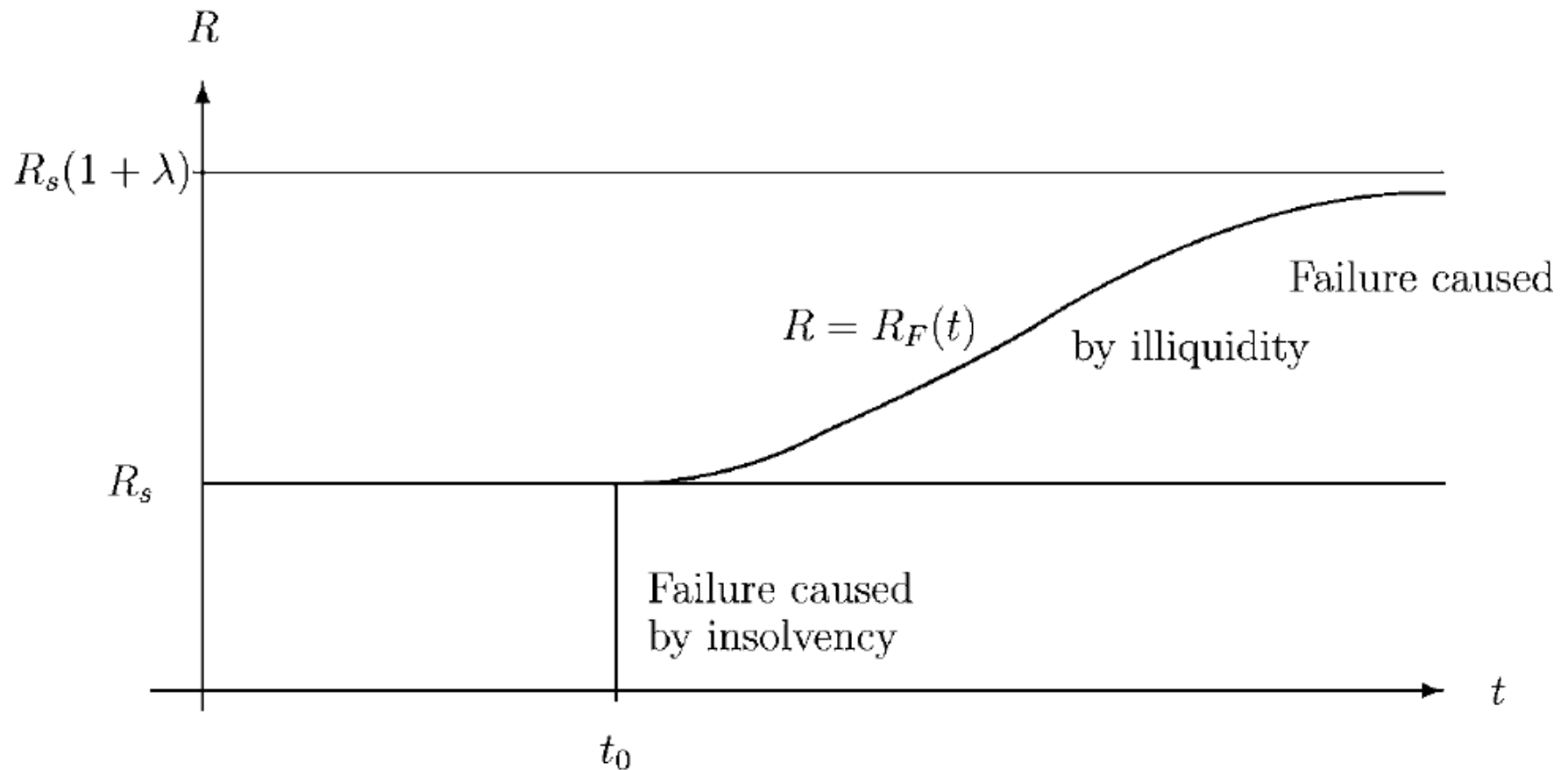


## EQUILIBRIUM OF THE INVESTORS' GAME

- ▷ How is  $x$  determined?
- ▷ without loss of generality, assume a threshold strategy for all managers
- ▷ withdraw if **SIGNAL**  $s < t$
- ▷ i.e. with proba:  $\mathbb{P}(R + \varepsilon < t) = F(t - R)$   
where  $F$  is the c.d.f of  $\varepsilon$
- ▷ this proba. also equals the proportion of withdrawals  $x(R, t)$
- ▷ moreover, we assumed that managers withdraw if
- ▷ the **PROBABILITY OF FAILURE**:  $P(s, t) > \gamma$
- $\Leftrightarrow \mathbb{P}(R < R_F(x(R, t)) \mid s) > \gamma \Leftrightarrow G(R_F(t) \mid s) \geq \gamma$   
where  $G(\cdot \mid s)$  is the c.d.f. of  $R$  conditional on signal  $s$

▷ now, as  $x(R, t) = F(t - R)$ ,  $R_F$  is implicitly defined by:

$$R = R_S \left( 1 + \lambda \left[ \frac{F(t-R) - k}{1-k} \right]_+ \right)$$



▷ with  $t_0/R_F = R_S$ , i.e.  $t_0 = R_S + F^{-1}(k)$   
 if  $t \geq t_0$ , "too many" failures  $\rightarrow$  need for a LOLR

## STRATEGIC COMPLEMENTARITY

▷ natural to assume  $G(r | s)$  decreasing in  $s$ :  
the higher  $s$ , the lower the proba that  $R < r$

⇒  $P(s, t)$  decreasing in  $s$ , increasing in  $t$   
( $P(s, t)$ : proba. of failure when signal  $s$  and threshold  $t$ )

⇒  $P(s, t) > \gamma \Leftrightarrow s < \bar{s}$  with  $\bar{s}/P(\bar{s}, t) = \gamma$ , i.e.  $\bar{s} = S(t)$   
with  $S'(t) = -\frac{\partial P/\partial t}{\partial P/\partial s} \geq 0$

⇒ a higher threshold  $t$  **BY OTHERS** induces a manager to use  
a **HIGHER THRESHOLD** also

## BAYESIAN EQUILIBRIUM (1)

- ▷ we look for a **STRATEGY** such that the equilibrium is consistent with the **BELIEFS**
- ▷ Managers withdraw if  $P(s, t) > \gamma$  and not withdraw if  $s < t$
- ▷ **CONSISTENT** iff  $t^*/P(t^*, t^*) = \gamma$
- ▷ then, as  $P(s, t)$  decreasing in  $s$ :
  - ▶  $s < t^* \Rightarrow P(s, t^*) > \gamma \Rightarrow$  withdraw
  - ▶  $s > t^* \Rightarrow P(s, t^*) < \gamma \Rightarrow$  not withdraw

## BAYESIAN EQUILIBRIUM (2)

▷ The equilibrium  $(R_F^*, t^*)$ , where

▶  $t^*$  is the equilibrium **WITHDRAWAL THRESHOLD**

▶  $R_F^*$  is the equilibrium **RETURN THRESHOLD**

is therefore determined by:

$$\begin{cases} G(R_F^* | t^*) = \gamma \\ R_F^* = R_S \left( 1 + \lambda \left[ \frac{F(t^* - R_F^*) - k}{1 - k} \right]_+ \right) \end{cases}$$

▷ 1st eq: if  $s = t^*$ ,  $\mathbb{P}(R < R_F^* | s) = \gamma$  (def of  $t^*$ )

▷ 2nd eq.: given  $t^*$ ,  $R_F^*$  is the return threshold, below which failure occurs (def of  $R_F^*$ )

## GAUSSIAN CASE

▷ to go further, we assume

$$\triangleright R \sim \mathcal{N}(\bar{R}, 1/\alpha)$$

$$\triangleright \varepsilon \sim \mathcal{N}(0, 1/\beta) \Rightarrow F(x) = \Phi(\sqrt{\beta}x)$$

▷ we look for  $G(R | s) = G(R | R + \varepsilon)$ . As

$$\blacktriangleright R + \varepsilon \sim \mathcal{N}(\bar{R}, 1/\alpha + 1/\beta), \text{ and}$$

$$\blacktriangleright \text{cov}(R, R + \varepsilon) = \text{Var}(R) = 1/\alpha$$

$$\triangleright \text{we have } R | R + \varepsilon \sim \mathcal{N}\left(\frac{\alpha\bar{R} + \beta s}{\alpha + \beta}, \frac{1}{\alpha + \beta}\right)$$

$$\triangleright \text{that is } G(R_F^* | t^*) = \Phi\left(\sqrt{\alpha + \beta}R_F^* - \frac{\alpha\bar{R} + \beta t^*}{\sqrt{\alpha + \beta}}\right)$$

## THE EQUILIBRIUM

▷ The equilibrium is then characterized

▷ by a pair  $(t^*, R_F^*)$  such that

$$\begin{cases} \Phi \left( \sqrt{\alpha + \beta} R_F^* - \frac{\alpha \bar{R} + \beta t^*}{\sqrt{\alpha + \beta}} \right) = \gamma \\ R_F^* = R_S \left( 1 + \lambda \frac{\Phi(\sqrt{\beta}(t^* - R_F^*)) - k}{1 - k} \right) \end{cases}$$

▷ and we can prove (proof omitted) that

**PROPOSITION.** When  $\beta$  (precision of private signal) large enough relative to  $\alpha$  (prior precision):

$$\beta \geq \frac{1}{2\pi} \left( \frac{\lambda \alpha D}{I} \right)^2 \equiv \beta_0$$

unique  $t^*$  such that  $P(t^*, t^*) = \gamma$ . The investor's game then has a unique equilibrium: a strategy with threshold  $t^*$ .



## COORDINATION FAILURE

▷ Failure caused by illiquidity (coordination failure) if  $t^* > t_0$

▷ with  $t^*$  such that:  $\Phi \left( \sqrt{\alpha + \beta} R_F^* - \frac{\alpha \bar{R} + \beta t^*}{\sqrt{\alpha + \beta}} \right) = \gamma$

▷ if  $t^* \leq t_0$ : **NO COORDINATION FAILURE**, i.e.  $R_F^* = R_S$ .

In this case:

$$t^* = \frac{1}{\beta} \left( (\alpha + \beta) R_S - \sqrt{\alpha + \beta} \phi^{-1}(\gamma) - \alpha \bar{R} \right)$$

▷ as  $t_0 = R_S + \frac{1}{\sqrt{\beta}} \phi^{-1}(k)$

▷ an equilibrium with  $t^* \leq t_0$  occurs iif:

$$(\alpha + \beta) R_S \leq \sqrt{\alpha + \beta} \phi^{-1}(\gamma) + \alpha \bar{R} + \beta R_S + \sqrt{\beta} \phi^{-1}(k)$$

## LIQUIDITY RATIO AND COORDINATION FAILURE

▷ That is iif:

$$k \geq \Phi \left( \frac{\alpha}{\sqrt{\beta}} (R_S - \bar{R}) - \sqrt{1 + \frac{\alpha}{\beta} \Phi^{-1}(\gamma)} \right) \equiv \bar{k}$$

**PROPOSITION.** There is a critical liquidity ratio  $\bar{k}$  of the bank such that, for  $k = \frac{K}{D} \geq \bar{k}$  **ONLY INSOLVENT BANKS FAIL** (there is no coordination failure).

▷ if  $k < \bar{k}$  solvent but illiquid banks fail

## PROBABILITY OF FAILURE

▷ In this last case  $R_F^*$  is defined by:

$$\begin{cases} \Phi \left( \sqrt{\alpha + \beta} R_F^* - \frac{\alpha \bar{R} + \beta t^*}{\sqrt{\alpha + \beta}} \right) = \gamma \\ R_F^* = R_S \left( 1 + \lambda \frac{\Phi(\sqrt{\beta}(t^* - R_F^*)) - k}{1 - k} \right) \end{cases}$$

$$\Leftrightarrow \begin{cases} -\sqrt{\alpha + \beta} \Phi^{-1}(\gamma) + (\alpha + \beta) R_F^* - \alpha \bar{R} - \beta t^* = 0 \\ t^* = R_F^* + \frac{1}{\sqrt{\beta}} \Phi^{-1} \left( \frac{1 - k}{\lambda R_S} (R_F^* - R_S) + k \right) \end{cases}$$

$$\Leftrightarrow \alpha (R_F^* - \bar{R}) - \beta \Phi^{-1} \left( \frac{1 - k}{\lambda R_S} (R_F^* - R_S) + k \right) - \sqrt{\alpha + \beta} \Phi^{-1}(\gamma) = 0$$

▷ As the l.h.s is decreasing in  $R_F^*$  for  $\beta \geq \beta_0$  we have

**PROPOSITION.**  $R_F^*$  – and therefore the proba of **FAILURE** – is decreasing in the liquidity ratio  $k$ , the critical withdrawal probability  $\gamma$ , and of the expected return  $\bar{R}$  and increasing in the fire-sale premium  $\lambda$  and the face value of debt  $D$ .

## HOW TO AVOID FAILURE CAUSED BY ILLIQUIDITY?

- ▷ theoretical possibility of a solvent bank being illiquid as a result of coordination failure on the interbank market.
- ▷ 2 possibilities (for a central bank or a gvt) to eliminate that:
  - ▶ lower bond on the liquidity ratio  $k$ :  $\bar{k}$
  - ▶ decrease  $\lambda$  through:
    - LIQUIDITY INJECTION** (as for ex after Sept 11)
    - DISCOUNT-RATE** lending (ex. Fed '08, low rate but stigma)

## DISCOUNT-RATE LENDING (1)

▷ fixing  $k \geq \bar{k}$ : costly in terms of "returns":

$I + K = 1 + E \Rightarrow$  high  $K$  means **LOW INVESTMENT**

▷ what to do if  $k < \bar{k}$ ?

▶ assume that the central bank lends at rate  $r \in (0, \lambda)$   
without limit, BUT only to **SOLVENT** banks

▶ Central bank not supposed to subsidize:  $r > 0$

▶ and assumed to perfectly observe  $R \leftarrow$  **SUPERVISION**

$\Rightarrow$  Optimal strat. for a bank = lend exactly  $D(x - k)_+$

$\rightarrow$  failure in  $\tau = 2$  iif

$$RI < (1 - x)D + (1 + r)(x - k)D$$

## DISCOUNT-RATE LENDING (2)

▷ That is, as  $R_S = \frac{D-K}{I} = \frac{D(1-k)}{I}$ , iff

$$R < R_S \left( 1 + r \frac{[x - k]_+}{1 - k} \right) \equiv R^*$$

(same as  $R_F^*$  with  $r$  instead of  $\lambda$ )

⇒ fully **EFFICIENT** ( $R^* = R_S$ ) if  $r$  arbitrarily close to 0

+ central bank **LOSES NO MONEY** (loan repaid at  $\tau = 2$ )  
as only lends to solvent banks ( $R > R_S$ )

⇒ possible!

## POSSIBLE EXTENSIONS

- ▷ including moral hazard
  - ▶ investment in risky assets requires supervision
  - ▶ supervision effort by bank manager  $e = \{0, 1\}$ ,  $e = 1$  costly
  - ▶  $e = 0 \Rightarrow R \sim \mathcal{N}(\bar{R}_0, \frac{1}{\alpha})$ ;  $e = 1 \Rightarrow R \sim \mathcal{N}(\bar{R}, \frac{1}{\alpha})$   
with  $\bar{R} > \bar{R}_0$
  - ▶ Result: the use of **SHORT-TERM** debt is optimal  
allowing withdraw at  $\tau = 1$  discipline bank managers
- ▷ **ENDOGENIZING**  $k = K/D$  (reserves chosen by the bank)

## INSURANCE, FAILURE AND RESERVES

see *Rees, Gravelle and Wambach, The Microeconomics of Insurance,*  
*section 3.2*

- ▷ insurance = **PROMISE** (against a premium)  
to pay coverage in case of accident
- ▷ how to make sure this promise is kept?  
i.e. the insurance has enough reserve to pay coverage?
- ▷ has to ensure insurance doesn't fail
- ▷ as banks: **CREDITORS** of insurance companies are policy-  
holders
- **CANNOT MONITOR** their insurance company
- ⇒ Existence of solvency **RULES** and **REGULATION** authorities



## THE MODEL

- ▷ an insurer offers a contract to  $n$  **IDENTICAL** individuals same risk (distribution of claims identical), same preferences
  - ▷ assume: **INDEPENDENT** risks ( $\rightarrow$  i.i.d.)  
not necessary to determine aggregate loss but simplifies
  - ▷  $\tilde{C}_i$  distrib of ind claims i.i.d.: mean  $\mu$  and variance  $\sigma^2$   
 $\Rightarrow \tilde{C}^n = \sum_{i=1}^n \tilde{C}_i$  distrib of aggregate claims, random var of mean  $n\mu$
- $\Rightarrow$  if premium sets to  $\mu$  ("fair" premium) on each contract and insurance costs are zero  
it will just **BREAK EVEN** ("rentable") in expected value:

$$\mathbb{E}(\text{Profit}) = \mathbb{E}(n\mu - \tilde{C}^n) = n\mu - \mathbb{E}(\tilde{C}^n) = 0$$

## THE NEED OF RESERVES

▷ However

$$\begin{aligned}\text{Var}(\text{Profit}) &= \text{Var}(\tilde{C}^n) = \mathbb{E} \left( \left( \sum_{i=1}^n \tilde{C}_i - n\mu \right)^2 \right) \\ &= \mathbb{E} \left[ \left\{ \sum_{i=1}^n (\tilde{C}_i - \mu) \right\}^2 \right] = \sum_{i=1}^n \mathbb{E} \left[ (\tilde{C}_i - \mu)^2 \right] \\ &= n\sigma^2\end{aligned}$$

is positive and linearly **INCREASING IN  $n$**

▷ no convergence:  $\forall n$ , we can have  $\tilde{C}_n <> n.\mu$

⇒ to **AVOID INSOLVENCY**

(when claims costs exceed funds available to meet them)

insurance have to carry **RESERVES**.

## RUIN PROBABILITY (1)

▷ reasonable to assume maximum cover  $C_{\max}$  per contract

⇒ maximum possible aggregate claims cost:  $nC_{\max}$

⇒ if premium  $P$  and reserves  $K_{\max} = n(C_{\max} - P)$ :

### ZERO PROBABILITY OF INSOLVENCY

▷ However, **IN PRACTICE**:

▶ proba. total claims near  $nC_{\max}$  extremely small

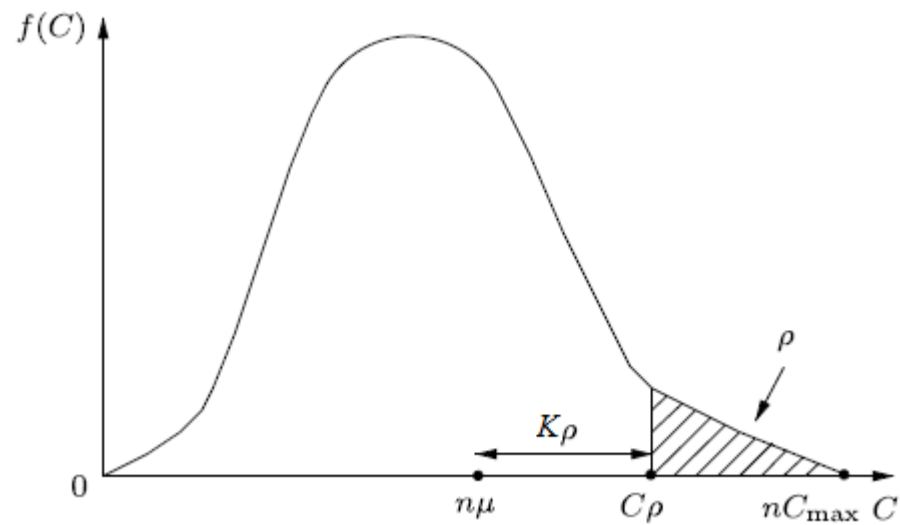
▶ raising capital of  $K_{\max}$  extremely costly

⇒ insurers choose a so-called **RUIN PROBABILITY**  $\rho$   
and given the distribution of  $\tilde{C}^n$  choose a level of reserves:

$$K(\rho) = C_\rho - nP \text{ with } C_\rho / \mathbb{P}(\tilde{C}^n > C_\rho) = \rho$$

## RUIN PROBABILITY (2)

- ▷ reserves / proba.  $\rho$  to be insolvent
- ▷ that is, when  $P = \mu$  (fair premium)



## HOW IS $\rho$ DETERMINED?

- ▷ Trade-off between
  - ▶ the costs associated with the **RISK OF INSOLVENCY** depends on buyers' **PERCEPTIONS** of this risk
  - ▶ and the cost of holding reserves
  
- ▷ explored in more detail in the **NEXT SECTIONS**

## THE IMPLICATIONS OF THE LAW OF LARGE NUMBERS

- ▷ let  $C_1, C_2, \dots, C_n$  the realizations of claims for  $n$  ind.  
(random sample from a distrib with mean  $\mu$  and var  $\sigma^2$ )
- ▷ let  $\overline{C}_n = \frac{1}{n} \sum_{i=1}^n C_i$  be the sample mean  
or the average **LOSS PER CONTRACT**
- ▷ Law of Large Numbers  $\rightarrow \forall \varepsilon > 0, \lim_{n \rightarrow \infty} \mathbb{P} (|\overline{C}_n - \mu| < \varepsilon) = 1$ :  
for sufficiently large  $n$ , virtually certain that the **LOSS PER CONTRACT** equals  $\mu$ , mean of individual loss distribution
- ▷ Moreover,  $\text{Var} (\overline{C}_n) = \mathbb{E} \left( \left( \frac{1}{n} \sum \tilde{C}_i - \mu \right)^2 \right) = \frac{1}{n} \sum_{i=1}^n \sigma^2 = \frac{\sigma^2}{n}$   
 $\Rightarrow$  the variance of realized loss per contract goes to 0 as  $n \rightarrow \infty$

## INTERPRETATION

- ▷ as the number of **CONTRACTS SOLD** becomes very large,
- ▷ risk that realized loss per contract exceeds fair premium becomes vanishingly small.

### ≈ **ECONOMY OF SCALE**

- ▶ although variance of aggregate claims increases with  $n$   
→ the reserves have to increase in absolute amount)
  - ▶ the required reserve per contract tends toward zero
- ▷ required reserves increase **LESS THAN PROPORTIONATELY** with size of the insurer (number of contracts)

## EXERCISE

- Consider a portfolio of 400,000 identical contracts for which
- ▷ the number of accidents per contract ( $N_i$ ) can be approximated by a  $\mathcal{P}(0, 07)$
  - ▷ the expected value of claims per accident  $\mathbb{E}(C_{ij}) = 14500 \text{ €}$
  - ▷ with a standard error  $\sigma(C_{ij}) = 130000 \text{ €}$
  - ▷  $N_i$  are assumed to be i.i.d
  - ▷ given  $N_i$ ,  $C_{ij}$  are assumed to be i.i.d  $\forall i, j$ 
    - ▶ Calculate the fair premium of a contract
    - ▶ Calculate the standard error of the annual claims on a contact
    - ▶ Calculate the amount of reserves that makes the ruin probability lower than 5%



## OPTIMAL CHOICE OF RESERVES

see *Rees and Wambach, The Microeconomics of Insurance, section 3.5*

- ▷ regulation: protect policyholders against risk of failure
- ▷ **REVERSE PRODUCTION CYCLE** → risk of fraud:  
once premiums paid insurer can **RUN OFF**
- ▷ **BUT** large, well-established companies
  - ▶ that wish to remain in business for the **LONG TERM**
  - ▶ would not need detailed regulatory intervention,
  - ▶ to ensure they carry enough reserves to meet obligations
- ▷ want to model these effects

## THE KEY ASSUMPTIONS

### 1. LIMITED LIABILITY ("engagement limité")

- ▶ a shareholder is **LIABLE** for the debts of a company **ONLY** up to the value of his shareholding

⇒ unregulated insurer may find optimal to put **NO RESERVES** and fail as soon as claims exceed collected premiums (more so if reserves are costly)

### 2. INCREASING FAILURE RATE

$$\frac{d}{dC} \frac{f(C)}{1 - F(C)} > 0$$

met by virtually all insurance loss-claims distributions

## THE MODEL (1)

- ▷ consider an insurance company in business for the long term
- ▷ so taking decisions over an **INFINITE TIME HORIZON**
- ▷ with a sequence of **DISCRETE TIME PERIODS** (say years).
- ▷ At the beginning of **EACH YEAR**
- ▷ decide on a level of reserve capital  $K$
- ▷ given the distribution of claims  $C$ :  $F(C)$   
with (differentiable) density  $f(C)$ , defined over  $[0, C_{max}]$
- ▷ **COSTLESS** reserves: owns (enough) capital but has to decide whether to invest or to commit it in the insurance business

## THE MODEL (2)

- ▷ premium income  $P$  **EXOGENOUS** (independent of  $K$ )  
buyers do not perceive relationship reserves and insolvency  
and act **AS IF NO SOLVENCY RISK**
- ▷  $P$  collected at the **BEGINNING** of the period  
and invested with  $K$  in **RISKLESS** asset (return  $r > 1$ )
- ⇒ At the end of the period assets:  $A = (P + K)r$ 
  - ▶ if  $A > C$ : **REMAINS IN BUSINESS**  
and receives continuation value  $V$  (expected present value  
of returns from insurance business over all future periods)
  - ▶ if  $A < C$ : **DEFAULTS**  
 $A$  used to pay claims, loses  $V$   
limited liability: doesn't pay claims above  $A$

## OPTIMAL RESERVES (1)

▷  $C < C_{max} \Rightarrow$  can always choose to guarantee solvency

▷ **QUESTION:** will insurers choose to stay solvent?

▷ it maximizes expected present value of future revenue

▷ i.e. chooses at each period  $K \in [0, K_{max}]$   
with  $K_{max} = \frac{C_{max}}{r} - P$  that

$$\max_K V_0(K) = \int_0^A \left( \frac{V}{r} + K + P - \frac{C}{r} \right) f(C) dC - K$$

(if solvent at  $t = 1$ :  $r(K + P) - C + V$ )

▷ **LIMITED LIABILITY**  $\Rightarrow$  upper limit  $A$

if  $C > A$ : insolvent, pays out  $A$ , loses  $V \Rightarrow$  integrand = 0

## OPTIMAL RESERVES (2)

▷ infinite horizon: future identical at begin. of each period  
⇒  $V = V_0(K)$  and:

$$V_0(K) = \left[ \int_0^A \left( K + P - \frac{C}{r} \right) f(C) dC - K \right] / \left[ 1 - \frac{F(A)}{r} \right]$$

▷ put another way: at each period, **IF SOLVENT**,  
i.e. with proba  $F(A)$  gets  $\left[ \int_0^A \left( K + P - \frac{C}{r} \right) f(C) dC - K \right]$   
next period (**DISCOUNTED** at rate  $1/r$ ):

$$V_0(K) = \sum_{t=0}^{+\infty} \left( \frac{F(A)}{r} \right)^t \left[ \int_0^A \left( K + P - \frac{C}{r} \right) f(C) dC - K \right]$$

## CORNER SOLUTION

**PROPOSITION.** If the claims distribution exhibits the increasing failure rate property then the solution of the optimization program of the insurer is a corner solution:

$$K = 0 \quad \text{or} \quad K = K_{max}$$

**PROOF:** There is no interior maximum:  
if  $\exists K^* \in (0, K_{max}) / V_0'(K^*) = 0$  then, under the assumption of increasing failure rate,  $V_0''(K^*) > 0$

## PROOF (1): FIRST ORDER CONDITION

$$V_0(K) = \left[ \int_0^{r(P+K)} \left( K + P - \frac{C}{r} \right) f(C) dC - K \right] / \left[ 1 - \frac{F(r(P+K))}{r} \right]$$

$$\begin{aligned} \Rightarrow V_0'(K) = & \frac{1}{\left(1 - \frac{F(A)}{r}\right)^2} \left( (-1 + F(A) + 0) \cdot \left(1 - \frac{F(A)}{r}\right) \right. \\ & \left. + f(A)V_0(K) \cdot \left(1 - \frac{F(A)}{r}\right) \right) \end{aligned}$$

$$\left( \frac{d}{dx} \int_0^{u(x)} f(x) dx = \int_0^{u(x)} f'(x) dx + f(u(x))u'(x) \right)$$

$$\Rightarrow V_0'(K^*) = \frac{1}{1 - \frac{F(A)}{r}} [V_0(K^*)f(A) - (1 - F(A))] = 0$$

where  $A = r(P + K)$



## PROOF (2): SECOND ORDER CONDITION

$$\Rightarrow V_0''(K) = \frac{1}{\left(1 - \frac{F}{r}\right)^2} \left( \left( V_0'(K)f + rV_0(K)f' + rf \right) \cdot \left( 1 - \frac{F(A)}{r} \right) + \left( V_0(K)f - (1 - F) \right) \cdot f \right)$$

$$= \frac{1}{\left(1 - \frac{F}{r}\right)} \left[ V_0'(K)f + rV_0(K)f' + rf + V_0(K) \cdot f \right]$$

$$\Rightarrow V_0''(K^*) = \frac{r}{\left(1 - \frac{F}{r}\right)} \left[ V_0(K^*)f' + f \right]$$

▷ what gives using the FOC  $V_0''(K^*) = \frac{r}{\left(1 - \frac{F}{r}\right)} \left[ \frac{(1-F)}{f} f' + f \right]$

▷ now the **ASSUMPTION** of increasing failure rate  $\frac{d}{dC} \frac{f(C)}{1-F(C)} > 0$  gives  $(1 - F)f' + f^2 > 0$

▷  $\nexists K^*/V'(K^*) = 0$  and  $V''(K^*) \leq 0$

## WHICH CORNER?

⇒ no interior solution

▷ but continuous function on  $(0, K_{max}) \Rightarrow \exists$  maximum

⇒ corner solution.

▷ Which corner? **COMPARE**  $V_0(0)$  and  $V_0(K_{max})$

$$V_0(0) = \frac{F(rP) (rP - \bar{C}_0)}{r - F(rP)}$$

$$V_0(K_{max}) = \frac{rP - \bar{C}}{r - 1}$$

with  $\bar{C} \equiv \mathbb{E}(C)$

and  $\bar{C}_0 \equiv \frac{1}{F(rP)} \int_0^{rP} C dF = \mathbb{E}(C \mid C \leq rP) < \bar{C}$

## COMPARISON

- ▷ **ADVANTAGE** not to put any reserve:
  - ▶ decrease expected claim costs ( $\bar{C}_0 < \bar{C}$ )
  - ▶ due to **LIMITED LIABILITY**
  
- ▷ **DISADVANTAGE**
  - ▶ risk  $1 - F(rP) > 0$  of going **OUT OF BUSINESS**
  
- ▷ In general, cannot say that a corner **ALWAYS BETTER**

## LIMITATIONS OF THE MODEL

- ▷ interest rate independent of the amount of capital raised
- ▷ no costs associated with raising capital
- ▷ **EXOGENEITY OF PREMIUM:** willingness to pay for insurance independent of insolvency risk
  - ▶ relaxed in the next model

## FAILURE RISK AND INSURANCE DEMAND:

see *Rees, Gravelle and Wambach, Regulation of Insurance Markets, GPRIT 1999*

- ▷ Assume now that policyholders **PERFECTLY OBSERVE** the reserves of their insurer
  - ▷ and can **INFER** from it its failure probability
  - ▷ First: simplest case of **JUST ONE** insurance buyer with income  $y$  (earned at end of period  $\rightarrow$  "borrow"  $P$ )  
loss distribution  $F(\cdot)$  on  $[0, C_u]$   
and utility function  $u(\cdot)$  with  $u' > 0$  and  $u'' < 0$
- $\Rightarrow$  in the absence of insurance: expected utility:

$$\bar{u}_0 \equiv \int_0^{C_u} u(y - C) dF$$

## INSURANCE DEMAND

- ▷ Assume insurer makes a "take-it-or-leave-it" offer
- ▷ "full cover" (repayment=loss) at a premium  $P$
- ▷ However, the buyer observes  $K$
- ▷ so the premium has to satisfy "participation constraint":

$$\int_0^A u(y - rP) dF + \int_A^{C_u} u(y - C - rP + A) dF \geq \bar{u}_0$$

- ▷ note  $P_0$  the **MAXIMAL PREMIUM** the buyer accepts when the insurer has **NO CAPITAL**:  $A = rP_0$ :

$$P_0 : F(rP_0)u(y - rP_0) + \int_{rP_0}^{C_u} u(y - C) dF = \bar{u}_0$$

- and  $P_u$  the **MAXIMAL PREMIUM** the buyer accepts when the insurer has **MAXIMUM CAPITAL**:  $A = C_u$ :

$$P_u : u(y - rP_u) = \bar{u}_0$$

## WHICH CORNER?

**PROPOSITION.** When the insurance buyer is fully informed about the insurer's choice of capital; the insurer's expected value is larger at  $(P_u, K_u)$  than at  $(P_0, K = 0)$ .

**PROOF:** we want to show that:

$$\frac{1}{r-1} \int_0^{C_u} (rP_u - C) dF > \frac{1}{r - F(rP_0)} \int_0^{rP_0} (rP_0 - C) dF$$

as  $r-1 < r - F$ , a sufficient condition would be

$$rP_u - \int_0^{C_u} C dF > F(rP_0)rP_0 - \int_0^{rP_0} C dF$$

$$\text{or } rP_u > F(rP_0)rP_0 + \int_{rP_0}^{C_u} C dF$$

## PROOF: JENSEN INEQUALITY (1)

▷ define  $\tilde{P}/u(y - r\tilde{P}) = \frac{1}{1-F(rP_0)} \int_{rP_0}^{C_u} u(y - C)dF$

▷ Jensen:  $u(\cdot)$  concave  $\Rightarrow \forall$  random var.  $\tilde{x} : u(\mathbb{E}(\tilde{x})) > \mathbb{E}(u(\tilde{x}))$

$$\Rightarrow r\tilde{P} > \frac{1}{1-F(rP_0)} \int_{rP_0}^{C_u} C dF \Rightarrow (1 - F(rP_0))r\tilde{P} > \int_{rP_0}^{C_u} C dF$$

▷ Moreover:

$$\begin{aligned} (1 - F(rP_0))u(y - r\tilde{P}) &= \int_{rP_0}^{C_u} u(y - C)dF \\ &= \bar{u}_0 - F(rP_0)u(y - rP_0) \\ &= u(y - P_u) - F(rP_0)u(y - rP_0) \end{aligned}$$

$$\Rightarrow u(y - rP_u) = F(rP_0)u(y - rP_0) + (1 - F(rP_0))u(y - r\tilde{P})$$



## PROOF: JENSEN INEQUALITY (2)

▷ Using again Jensen's inequality, we have:

$$rP_u > F(rP_0)rP_0 + (1 - F(rP_0))r\tilde{P}$$

▷ what implies using previous result that:

$$rP_u > F(rP_0)rP_0 + \int_{rP_0}^{C_u} C dF$$

Q.E.D

(a similar result can be proved for any  $K < K_u$ )

## INTUITION

- ▷ Due to risk aversion ( $u(\cdot)$  concave)
- ▷ policyholder always prepared to pay more than fair premium
- ▷ to **INSURE AGAINST INSURER'S INSOLVENCY**
- ⇒ the insurer (risk-neutral) gains at selling this
- ⇒ he must put up enough capital to **REMAIN SOLVENT**

(For now only shown in the simple case of only one buyer)

## GENERALIZATION TO $N$ POLICYHOLDERS

▷ need more assumptions

▶ on individual risk

▶ on **HOW  $A$  IS SHARED** in case of failure

▷ we assume

▶ i.i.d risk of losing  $L(< y)$  with proba  $p$ :  $C \sim \mathcal{B}(n, p)$

▶ in case of failure by the insurer, each policyholder

• receive indemnity in full w/ proba  $A/C$

• receive nothing with proba  $(1 - A/C)$

## PARTICIPATION CONSTRAINT

▷ a policyholder **WILLING TO PAY**  $P$  for full coverage if:

$$(1 - p)u(y - rP) + p \left\{ (1 - \pi)u(y - rP) + \pi \left[ (1 - \theta)u(y - rP) + \theta u(y - rP - L) \right] \right\} \geq \bar{u}_0$$

with  $\pi$ : proba insurer insolvent given he suffers the loss

and  $\theta$ : proba he receives nothing in this case

▷ that is, noting  $q \equiv p\pi\theta$

$$(1 - q)u(y - rP) + qu(y - rP - L) \geq \bar{u}_0$$

## RESERVE AND FAILURE

▷ Suppose insurer chooses reserves to **MEET A GIVEN NUMBER**  $n < N$  of losses. Then:

$$q = p \sum_{m=n-1}^{N-1} \binom{N-1}{m} p^m (1-p)^{N-1-m} \left(1 - \frac{n}{m+1}\right)$$

▷ can then prove equivalent result to previous Proposition

**PROPOSITION.** If buyers know the probability  $q$  that they will not be compensated, the insurer maximizes his expected value by choosing a capital  $K_m$  so that there is no default risk ( $q = 0$ ).

$$K_m = \frac{N(L-rP_m)}{r} \quad \text{w/} \quad P_m \text{ largest acceptable premium for } q = 0$$

## PROOF (SIMILAR)

▷ we want to show that,  $\forall q > 0$

$$\frac{1}{r-1}N(rP_m - pL) > \frac{1}{r-(1-d)}N(rP_q - (p-q)L)$$

w/  $d$ : default proba;  $P_q$ : largest acceptable premium for  $q$

▷ as  $q > 0 \Rightarrow r-1 < r-(1-d)$ , sufficient to show that

$$rP_m \geq rP_q + qL$$

▷ by definition:

$$u(y - rP_m) = (1-q)u(y - rP_q) + qu(y - rP_q - L) = \bar{u}_0$$

▷ and Jensen's inequality gives:

$$rP_m > (1-q)rP_q + q(rP_q + L) = rP_q + qL$$

**Q.E.D**

## INTUITION (SIMILAR)

- ▷ policyholders always willing to pay more **THAN THE FAIR PREMIUM**
- ▷ to insure against **INSURER'S INSOLVENCY**,
- ▷ the insurer finds it **PROFITABLE** to sell him this
- ▷ but **REQUIRES** to put enough capital to remain solvent

## CONCLUSIONS

- ▷ if policyholders **NAIVELY** believe that the their insurer would **REMAIN SOLVENT**
  - ▶ might be optimal for insurers **NOT TO HOLD RESERVES** and to bear **FAILURE RISK**
- ▷ **BUT** if policyholders **PERFECTLY INFORMED** about insurers failure risk
  - ▶ always optimal for insurers to reduce this **RISK TO ZERO**
- ⇒ **PRINCIPE OF REGULATION**: provide policyholders w/ information about insurers failure risk
  - ▶ **DISCLOSURE** on capital, risk exposure,...
  - + minimal capital requirement  $\approx$  maximal failure proba



## LIMITATIONS OF THE MODEL

- ▷ interest rate independent of the amount of capital raised
- ▷ no costs associated with raising capital
- ▷ impossibility to **RECAPITALIZE** at the end of each period  
**AFTER** claims realization, if  $A < C$ , insurer might want to raise some capital to **REMAIN SOLVENT**

## ALLOWING FOR RECAPITALIZATION

see *Bourlès and Henriët, 2009*

- ▷ Recall: Why to regulate?
  - ▶ asymmetric information → solution = **DISCLOSURE**
  - ▶ conflict of **INTEREST** betw/ shareholders & policyholders
- ▷ for the insurer to fail:
  - ▶ not only **RESERVES** has to be **INSUFFICIENT**
  - ▶ but also has to be **SUBOPTIMAL** to recapitalize
- ▷ Including shareholders in the model, new choices:
  - ▶ if solvent: take **DIVIDEND** or increase reserves (new shares)
  - ▶ if insolvent: failure or **RECAPITALIZE** (increase reserves)
- ⇒ information on reserve **NOT SUFFICIENT**
- ▷ failure also depends on recap policy ⇒ **CREDIBILITY ISSUE**

## FULL COMMITMENT

- ▷ In such a model, the insurance company has to choose
  - ▶ how much capital it holds ( $K$ )
  - ▶ a **RECAPITALIZATION POLICY**:  
the interval of claims that will be indemnified ( $I$ )
  - ▶ an **ISSUANCE AND DIVIDEND POLICY**
- ▷ moreover assume that capital is **COSTLY**:  
return on reserves lower than interest rate
- ▷ From previous analysis:
  - ▶ if insurer can **COMMIT EX-ANTE** on a recap. policy
  - ▶ it commit **NEVER TO DEFAULT**
  - ▶ costly capital  $\rightarrow K = 0, I = [0, +\infty)$

## NO COMMITMENT

- ▷ If insurer cannot **CREDIBLY COMMIT** on  $I$ , **EX-POST**:
  - ▶ insurer optimally default if amount needed to continue
  - ▶ is larger than the present value of the insurance company
  
- ▷ When reserves are **UNOBSERVABLE**, we can show that
  - ▶ insurer never holds reserves:  $K^* = 0$
  - ▶ shareholders take **DIVIDENDS** as soon as possible (never leave money in the insurance company)
  - ▶ failure occurs optimally when claims exceed the value of the company

## NO COMMITMENT - OBSERVABLE RESERVES

- ▷ When reserves are **OBSERVABLE**
    - ▶ optimal to hold reserves:  $K^* > 0$
    - ▶ as it increases the maximal acceptable premium
    - ▶ failure occurs optimally when claims exceed the value of the company
    - ▶ **BUT**: threshold higher than in previous case:  
higher premium → **HIGHER VALUE**
- ⇒ **LOWER PROBA OF FAILURE**

## IMPLICATIONS FOR REGULATION

- ▷ **INFORMATION DISCLOSURE** gets part of the way
- ▷ **RESERVE REQUIREMENT** can also be useful:  
by  $\uparrow$  the value of the company, it  $\downarrow$  the probability of failure
- ▷ But best regulation would be
  - ▶ **TO MAKE CREDIBLE** the commitment to always recap.
  - ▶ for ex. by setting a **GUARANTEE FUND**
  - ▶ but... would introduce **MORAL HAZARD** for shareholders  
(no incentives to hold reserves)

## INSURANCE REGULATORY FRAMEWORK: SOLVENCY I

- ▷ "Current" European regulation: Solvency I
  - ▶ established in 1973, amended in 2002
  - ▶ **SOLVENCY MARGIN REQUIREMENTS (SMR)**
  - ▶ financial guarantee in addition to provisions
  - ▶ reserves  $>$  SMR = 4% of provisions + 3‰ of capital at risk
  
- ▷ **SIMPLE AND ROBUST** framework BUT
  - ▶ no "true" **MEASURE OF RISK** taken by the insurer
  - ▶ no **QUALITATIVE** requirement (quality of data)
  - ▶ no **DIVERSIFICATION** effect
  - ▶ no role for **INFORMATION**

## INSURANCE REGULATORY FRAMEWORK: SOLVENCY II

- ▷ New European regulation: Solvency II
  - ▶ Reform adopted in 2009 by the European Parliament
  - ▶ scheduled to come into effect on 1 January 2016  
(after 01/01/13 and 01/01/14...)
  
- ▷ Relies as Basel accords on 3 pillars:
  - ▶ Pillar I: **QUANTITATIVE** requirements
  - ▶ Pillar II: **QUALITATIVE** requirements
  - ▶ Pillar III: **DISCLOSURE** and transparency requirements



Pillar I <b>QUANTITATIVE REQUIREMENT</b>	Pillar II <b>QUALITATIVE REQUIREMENT</b>	Pillar III <b>DISCLOSURE REQUIREMENT</b>
<ul style="list-style-type: none"> <li>▷ Asset evaluation</li> <li>▷ Risk definition</li> <li>▷ Evaluations of               <ul style="list-style-type: none"> <li>▶ technical provisions</li> <li>▶ "target" capital (SCR)</li> <li>▶ minimum capital (MCR)</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>▷ Internal control</li> <li>▷ Risk management</li> <li>▷ Reinforcement and harmonization of external control at EU level</li> </ul>	<ul style="list-style-type: none"> <li>▷ Requirement for standardized information for market authority regulators, investors and policyholders</li> <li>▷ transparency of financial reporting</li> </ul>

## TWO LEVELS OF CAPITAL REQUIREMENT

- ▷ SCR (Solvency Capital Requirement)
  - ▶ capital required to ensure that insurance company able
  - ▶ to **ABSORB SIGNIFICANT UNEXPECTED EVENTS** (bicentennial event)
  - ▶ and **GUARANTEE SOLVENCY** in face of such events
  - ▶ If capital < SCR: insurance is required to ↑ capital
  - ▶ **TARGETED** value of capital
- ▷ MCR (Minimal Capital Requirement)
  - ▶ level for which insurer's activity pose an
  - ▶ **UNACCEPTABLE RISK** to policyholders
  - ▶ If capital < MCR: license withdrawn & liabilities transferred to another insurer

## HOW IS THE SCR CALCULATED?

### ▷ RISK MEASURE

#### ▶ V@R: VALUE AT RISK

▶ Potential loss to be suffered on a portfolio over a given period with a given probability  $\alpha$

▶ = quantile of loss-and-profit distribution  $X$  (asset variation; in our model  $X = nP - C$ ):

$$\mathbb{P} ( V@R_{1-\alpha} < X ) = \alpha$$

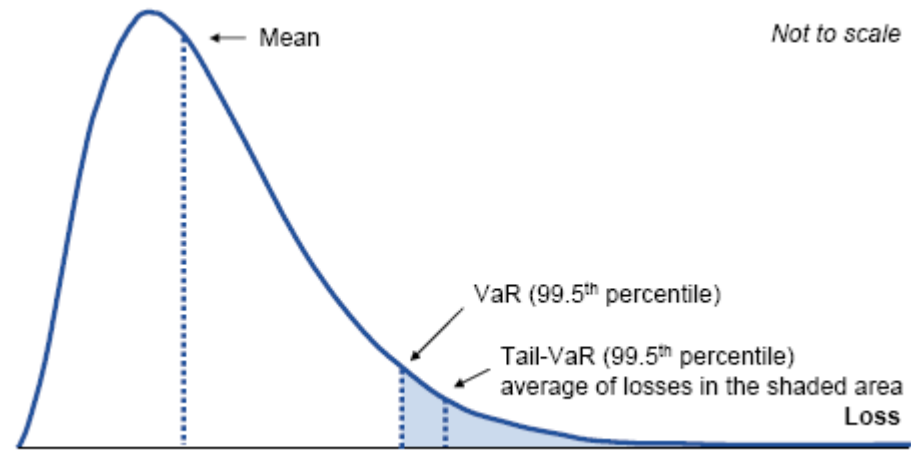
### ▷ CALIBRATION of the SCR

▶ SCR = Value-at-Risk at 99.5% over 1-year

▶ failure probability on 1 year  $< 0.5\%$

▶ able to absorb bicentennial (adverse) event

## VALUE AT RISK AT 99.5%



Is it a good measure of risk?