Theory of incentives

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Related textbooks

- Bolton, P. and Dewatripont, M., *Contract Theory*, MIT Press.
- Laffont, J.-J. and D. Martimort, D., *The Theory of Incentives – The Principal-Agent Model*, Princeton University Press.
- Salanié, B., *The Economics of Contracts: A Primer*, MIT Press.

1 Introduction: information, incentives and contracts

The basic aim of the theory of incentives is to deal with asymmetric (or decentralized) information in economic interactions, in particular when the objectives of the various parties are conflicting. It allows both to deal with regulatory issues generally absent form general equilibrium models and to analyze more precisely the inner working of firms. Basic examples cover the provision of public goods (when the government has imperfect information on preferences) and the issue of task delegation, be it in the case of a manager delegating a task to a worker, or of a government delegating the operation of a natural monopoly to a firm. In these cases, unobserved actions or private knowledge about cost or valuation, make the problem deviate from classical models. In particular, welfare theorems often fail to hold and efficiency cannot be reached. This is mostly caused by the necessity to "offer" informational rents to the agent detaining the information. Other classical examples highlighting the role of theory of incentives cover optimal taxation, price discrimination, auctions or insurance.

Among asymmetries of information we will distinguish two situations: hidden information and hidden action. In the first case, the parties are unequally informed about the characteristics of the relationship. The relevant information might concern for example the cost or the efficiency of production, the buyer's willingness to pay for the product, or the riskiness of an action or an investment. The models of this class will be referred to as adverse selection models, because in such situations not distinguishing between the various "types" would lead the uninformed party to mostly deal with the worse types, from its point of view. In the case of hidden action, the asymmetry of information concerns a choice, often called effort, made by the informed party(ies). The issue of incentives is then for the uninformed party(ies) to make the informed one(s) chose a level of effort in line with its (their) objective(s). We will refer to these situations as "moral hazard".

In most of the lectures, we will place ourselves in a Principal-Agent model, that is in a situation (i) with only two parties: one informed and one uninformed, in which (ii) one of the two parties called the Principal makes a take-it-or-leave-it offer to the other party called the

Agent. We will therefore assume away the issue of bargaining between the parties (that is game theoristic considerations[1]) as well as the issue of contract enforceability (every ex-ante agreement will be assumed to be ex-post enforced).

The course is organized as follows. We consecutively present the basic models of adverse selection and moral hazard, their main applications, some extensions and the dynamic aspects. We then turn to the limits of the theory of incentives, *i.e.* when they work badly, through the concept of countervailing incentives and behavioral aspects.

2 Hidden information: screening and signaling (R. Bourlès)

Let us consider first the case of hidden information. We will analyze separately situations in which the information is owned by the Principal and by the Agents, starting will the former.

We consider in the next three subsections cases in which the Agent owns private information over one of his characteristics that is relevant for Principal's welfare. As stated before, such situations will often be referred to as "adverse selection". This is because, most a the time, the consequences of the Principal ignoring this private information, is favoring the agents that provides her with the worst welfare.[2] A silent example is the one of insurance: offering a price (or premium) based on average risk exposure favors agents with higher-than-average risk, that are the more costly for the insurer. To escape from this adverse selection issue, the Principal needs to find a way to provide different Agent types with different allocations. We refer to this process as "screening". One of the most classic example of screening is second-degree price discrimination.

2.1 A classic example: recall on second degree price discrimination

(see e.g. Salanié section 2.2)

Consider a wine seller, the Principal, who face a potential buyer, the Agent. The Agent may be of two types: either he is "sophisticated" and is ready to pay a high price for good vintage; or he is "frugal" and has less developed taste for wine. We assume that the Agent know is type, but that the Principal doesn't.[3]

We assume more specifically that the utility of an agent of type i ($i \in \{1, 2\}$) writes

$$U_i(q, p) = \theta_i q - p$$

if he buys a bottle of quality q at price p; and that θ_i can take two values $\theta_1 < \theta_2$ (the sophisticated agent is then the agent of type 2).

In our example, the principal is a local monopolist (as he is able to make a take-it-or-leave-it offer) and we assume that she produces a bottle of quality q at cost $C(q)$, with $C'(\cdot) \geq 0$, $C''(\cdot) > 0$, $C'(0) = 0$ and $\lim_{q \to +\infty} C'(q) = +\infty$. Her profit on a bottle of quality q sold at price p thus writes

$$\Pi(q, p) = p - C(q)$$

[1] In our setting with asymmetric information, perfect Bayesian equilibrium would be the natural concept.

[2] We will often refer to the Principal as "she" and to the Agent as "he"

[3] This example also easily fits the insurance market, the credit market (in which buyers are likely to better know their risk exposure than the seller) or the labor market.

Therefore if the Principal would observe the type of each Agent, she will offer to an Agent of type i a bottle of quality q_i at price p_i satisfying:

$$\begin{array}{ll} \max\limits_{p_i, q_i} & p_i - C(q_i) \\ s.c. & \theta_i q_i - p_i \geq 0 \end{array}$$

that is q_i^* such that $C'(q_i^*) = \theta_i$ and $p_i^* = \theta_i q_i^*$. She will therefore sell efficient qualities (the social surplus indeed writes $\theta_i q - C(q)$ for a bottle sold at an Agent of type i) and extract all the surplus from the transaction $(U_i(q_i^*, p_i^*) = 0\ \forall i)$.

Now assume that the Principal doesn't observe the Agent type and only knows the proportion of each type in the population. We denote by α the proportion of type-1 Agents. Let's assume first (and prove in the next section) that the Principal's best strategy is to offer to every Agent a menu with two bottles of different qualities at different prices.

Note first that, if the Principal offers the above "first-best" menu (q_1^*, p_1^*), (q_2^*, p_2^*), sophisticated (*i.e.* type-2) Agents would prefer the "frugal" option as:

$$U_2(q_1^*, p_1^*) = (\theta_2 - \theta_1) q_1^* > 0 = U_2(q_2^*, p_2^*)$$

This means, in particular, that if the Principal wants to sell both types of bottle, she has to leave some surplus to type-2 Agents. We would call this surplus "informational rent". It also means that the contracts (quality,price) offered to each type cannot be set independently anymore.

More precisely, if the Principal wants the two types of bottle to be sold, the contracts $\{(q_1, p_1), (q_2, p_2)\}$, *i.e.* the menu, has to solve the two following constraints:

$$\left\{ \begin{array}{lcl} U_1(q_1, p_1) & \geq & U_1(q_2, p_2) \\ U_2(q_2, p_2) & \geq & U_2(q_1, p_1) \end{array} \right. \Leftrightarrow \left\{ \begin{array}{lcl} \theta_1 q_1 - p_1 & \geq & \theta_1 q_2 - p_2 \\ \theta_2 q_2 - p_2 & \geq & \theta_2 q_1 - p_1 \end{array} \right.$$

that we call the incentive (or self-selection) constraints.

Assuming that the Principal maximizes her expected profit, the problem becomes:

$$\begin{array}{lll} \max\limits_{p_1, q_1, p_2, q_2} & \alpha\,(p_1 - C(q_1)) + (1-\alpha)\,(p_2 - C(q_2)) & \\ s.t. & \theta_1 q_1 - p_1 \geq 0 & (\mathrm{PC}_1) \\ & \theta_2 q_2 - p_2 \geq 0 & (\mathrm{PC}_2) \\ & \theta_1 q_1 - p_1 \geq \theta_1 q_2 - p_2 & (\mathrm{IC}_1) \\ & \theta_2 q_2 - p_2 \geq \theta_2 q_1 - p_1 & (\mathrm{IC}_2) \end{array}$$

Now,

- as $\theta_2 > \theta_1$, (PC_2) can be neglected: it is implied by (IC_2) and (PC_1)

$$\theta_2 q_2 - p_2 \geq \theta_2 q_1 - p_1 > \theta_1 q_1 - p_1 \geq 0$$

- one can then prove by contraction that (IC_2) is binding: if not, the Principal could increase p_2 thereby increasing its objective without breaking any constraint. Then, at the optimum

$$p_2 - p_1 = \theta_2(q_2 - q_1) \tag{1}$$

- adding (IC_1) and (IC_2) gives $\theta_2(q_2 - q_1) \geq \theta_1(q_2 - q_1)$ and therefore $q_2 \geq q_1$ (as $\theta_2 > \theta_1$). Then, (IC_2) being binding one can neglected (IC_1):

$$p_2 - p_1 = \theta_2(q_2 - q_1) > \theta_1(q_2 - q_1)$$

- as above, once (IC_1) neglected, one can prove by contradiction that (PC_1) is binding: if not, the Principal could increase p_1 thereby increasing its objective without breaking any constraint. Then, at the optimum

$$p_1 = \theta_1 q_1 \tag{2}$$

Using (1) and (2), the problem of the principal then becomes:

$$\max_{q_1, q_2} \ \alpha\left(\theta_1 q_1 - C(q_1)\right) + (1-\alpha)\left(\theta_1 q_1 + \theta_2(q_2 - q_1) - C(q_2)\right) \tag{3}$$

that gives, denoting the solutions $\tilde{q}_1$ and $\tilde{q}_2$:

$$C'(\tilde{q}_2) = \theta_2 \tag{4}$$

$$C'(\tilde{q}_1) = \theta_1 - \frac{1-\alpha}{\alpha}(\theta_2 - \theta_1) < \theta_1 \tag{5}$$

and by (1) and (2): $\tilde{p}_1 = \theta_1 \tilde{q}_1$ and $\tilde{p}_2 = \theta_2 \tilde{q}_2 - (\theta_2 - \theta_1)\tilde{q}_1$.

Then, a type-2 Agent is offered efficient quality (as in the full information case) and – due to asymmetric information – gets a positive surplus: $U_2(\tilde{q}_2, \tilde{p}_2) = \theta_2 q_2 - p_2 = \tilde{q}_1(\theta_2 - \theta_1) > 0$ that we denote informational rent (note that he is then by construction indifferent between the two bottle).[4]

On the contrary, a type-1 agent gets less than efficient quality (as $C(\cdot)$ is convex) and no surplus $U_1(\tilde{q}_1, \tilde{p}_1) = \theta_1 \tilde{q}_1 - \tilde{p}_1 = 0$. Note moreover that the quality offered to a type-1 agent is decreasing with the proportion of sophisticated agents and with the difference in taste between the two types of agents. Indeed, the issue is here for the Principal, the seller, to make the sophisticated agent buy the high-price bottle. To do so, she (i) leaves him a informational rent (a Sophisticated agent can always pretend to be a Frugal one a get positive surplus, whereas the reverse is not true) and (ii) decreases the quality of the low-price bottle for it to be less attractive to a Sophisticated agent.

Some of the above patterns can be generalized to the n-type case: $\theta_n > \theta_{n-1} > \ldots > \theta_1$. Then, one can show that:

- The highest type gets efficient quality. One says we keep "efficiency at the top".
- All types but the lowest gets positive surplus. It is their informational rent. This surplus increases with type.

[4]One can rewrite the objective as $\alpha\left(\theta_1 q_1 - C(q_1)\right) + (1-\alpha)\left(\theta_2 q_2 - C(q_2)\right) - (1-\alpha)q_1(\theta_2 - \theta_1)$, that is the full information objective minus the expected informational rent that has to be given to a type-2 Agent.

Note that $(\tilde{q}_1, \tilde{q}_2)$ from (5) and (4) are indeed the solutions of (3) only if they are positive. If $\theta_1 - \frac{1-\alpha}{\alpha}(\theta_2 - \theta_1) < 0$, as $C'(0) = 0$ and $C''(\cdot) > 0$, (5) has no solution and q_1 is optimally set to 0. In other terms, type-1 agents are excluded from the market. This notably happens when α is low, that is when they are only few Frugal agents in the population. In this case, the Principal prefers to sell only high-quality wine to Sophisticated agents, and do not need to leave them any informational rent (they cannot mimic Frugal agents anymore). One can indeed easily see that when $q_1 = 0$: $\tilde{p}_2 = \theta_2\tilde{q}_2 = \theta_2 q_2^* = p_2^*$. By excluding Frugal agents, the Principal can offer Sophisticated agents the same bottle as under full information.

When $\tilde{q}_1 \geq 0$, she would prefers to do so if:

$$
\begin{aligned}
& (1-\alpha)(p_2^* - C(q_2^*)) > \alpha(\tilde{p}_1 - C(\tilde{q}_1)) + (1-\alpha)(\tilde{p}_2 - C(\tilde{q}_2)) \\
\Leftrightarrow\; & (1-\alpha)(\theta_2 q_2^* - C(q_2^*)) > \alpha(\theta_1\tilde{q}_1 - C(\tilde{q}_1)) + (1-\alpha)(\theta\tilde{q}_2 - (\theta_2-\theta_1)\tilde{q}_1 - C(\tilde{q}_2)) \\
\Leftrightarrow\; & (1-\alpha)(\theta_2 q_2^* - C(q_2^*)) > \alpha(\theta_1\tilde{q}_1 - C(\tilde{q}_1)) + (1-\alpha)(\theta q_2^* - (\theta_2-\theta_1)\tilde{q}_1 - C(q_2^*)) \\
\Leftrightarrow\; & C(\tilde{q}_1) > \left(\theta_1 - \frac{1-\alpha}{\alpha}(\theta_2 - \theta_1)\right)\tilde{q}_1
\end{aligned}
$$

that is if $C(\tilde{q}_1) - C'(\tilde{q}_1)\tilde{q}_1 > 0$. This never holds when $C(\cdot)$ is convex and $C'(0) = 0$. Thus, the Principal always prefers not to exclude Frugal consumers when $\tilde{q}_1 \geq 0$.

2.2 Mechanism design and revelation principle

(see e.g. Salanié section 2.1)

We have assumed in the previous section that it was optimal for the Principal to offer a menu of two contracts. We will show now that it is indeed the case and notably that the Principal cannot achieve a better outcome by offering more options than the number of types; nor by using direct communication with the Agent. This would be a consequence of the so-called revelation principle, which is one of the basic result of mechanism design.

The aim of mechanism design is basically to build (allocation and communication) rules that allow the achievement of a specific outcome, when the relevant information is dispersed among economic agents. The most classical example is the provision of public good, in which a government would like to know the willingness to pay of all the citizens for a given public good it envisages to build. When these willingness to pay are private information, the way the decision (to build or not) is made and how the costs are shared crucially determine what each citizen declares.

More generally, mechanism design will allow to deal with situations in which n agents $i = 1, ..., n$ characterized by their type $\theta_i \in \Theta$, which are private information; face a Principal whose aim is to implement a given allocation that depends on agents' private information θ_i.

To do so, the Principal builds up what we call a mechanism $(g(\cdot), \mathcal{M}_1, ..., \mathcal{M}_n)$ consisting of a message space $\mathcal{M}_i$ for each agent i and a function $g(\cdot)$ from $\mathcal{M}_1 \times ... \times \mathcal{M}_n$ to the set of feasible allocations. The allocation rule $g(\cdot) = (g_1(\cdot), ..., g_n(\cdot))$ determine the allocation to each agent depending on all the messages. Given these rules and his preferences, each agent sends a message $m_i \in \mathcal{M}_i$ and the allocation $g(m_1, ..., m_n)$ is implemented. In the case of the public good, a possible mechanism consists of the rules stating under which condition the good is built and how it will be financed; and an agent message would consist in his willingness to pay.

In our Principal-Agent case, that is with only one agent, the message sent by the agent will depend on the mechanism $(g(\cdot), \mathcal{M})$ and his private information:

$$m^*(\theta) \in \arg\max_{m \in \mathcal{M}} u(g(m), \theta)$$

and he will the obtain $g^*(\theta) = g(m^*(\theta))$.

The revelation principle states in this case that:

> Any allocation $g^*(\theta)$ obtained with a mechanism $(g(\cdot), \mathcal{M})$ can also be implemented with a mechanism that is both **direct** (in which the set of message is the set of types) and **truthful** (in which the agent finds it optimal to report his true type).

By this principle one can therefore restrict attention to direct and truthful mechanisms.

The revelation principle can be easily proven in our simple one-agent case. Let $(g(\cdot), \mathcal{M})$ be a mechanism leading to allocation g^*, with equilibrium message $m^*(\theta)$. By composition ($g^* = g \circ m^*$), one can always compute the corresponding mechanism $(g^*(\cdot), \Theta)$. Now, by definition of equilibrium message:

$$\begin{aligned} & m^*(\theta) \in \arg\max_{m \in \mathcal{M}} u(g(m), \theta) && \forall \theta \in \Theta \\ \Leftrightarrow \quad & u(g(m^*(\theta), \theta) \geq u(g(m^*(\theta'), \theta) && \forall (\theta, \theta') \in \Theta^2 \end{aligned}$$

that is by definition of $g^*(\cdot)$:

$$u(g^*(\theta), \theta) \geq u(g^*(\theta'), \theta) \quad \forall (\theta, \theta') \in \Theta^2$$

meaning that the direct mechanism must be truthful.

In our example of wine seller, the allocation g consists of a quality q and a price p. The revelation principle states that in this case, to implement quality $q(\theta)$ using price $p(\theta)$, it is enough to offer a menu of contract with $|\Theta|$ options. The agent will then reveal his type θ, receive $q(\theta)$ and pay $p(\theta)$. In this case of menus, although the mechanism is direct and trustful, the messages are not explicit, as the buyer doesn't say neither "I am Sophisticated" nor "I am Frugal".

2.3 A more general model of adverse selection

(see e.g. Salanié section 2.3)

Let us now consider a more general continuous version of adverse selection in which the Principal and the Agent exchange a quantity of good q at a (total) price p. We assume that the agent's type θ (his private information) is drawn from a continuous set $[\underline{\theta}, \overline{\theta}]$ and that, as above, it doesn't enter directly the Principal's objective. We denote by $\Pi(q,p) \equiv p - C(q)$ the objective of the principal and by $U(q,p,\theta) \equiv u(q,\theta) - p$ the objective of a type-θ Agent, with $u(\cdot)$ increasing in both arguments.

Regarding the information sets, we assume that the agent knows his type before the contract is signed[5] and that the Principal only knows the distribution of types in the population (we denote by $F(\cdot)$ this distribution on $\Theta = [\underline{\theta}, \overline{\theta}]$ and $f(\cdot)$ the corresponding density), that we will call her prior.

We know from the above revelation principle that it is then enough for the Principal to offer a menu of contract $(q(\cdot), p(\cdot))$ that depends on the revelation of Agent's type θ, which should be truthful at the equilibrium. As above, this menu has thus to verify for each type of Agent:

- Incentive constraints: a type-θ agent must optimally choose the contract $(q(\theta), p(\theta))$ the Principal designed for him
- Participation constraints: at $(q(\theta), p(\theta))$, a type-θ agent must earn a utility level larger than his reservation utility (*i.e.* larger than the one reached with his outside option)

Let us start with the incentive constraints. If we denote by $\hat{\theta}$ the type announced by the agent and define

$$V(\theta, \hat{\theta}) \equiv U(q(\hat{\theta}), p(\hat{\theta}), \theta)$$

incentive constraints can be written as

$$\forall \theta \in \Theta, \;\; \theta \in \arg\max_{\hat{\theta} \in \Theta} V(\theta, \hat{\theta})$$

that is:

$$\forall \theta \in \Theta, \begin{cases} \frac{\partial V}{\partial \hat{\theta}}(\theta, \theta) = 0 \\ \frac{\partial^2 V}{\partial \hat{\theta}^2}(\theta, \theta) \leq 0 \end{cases}$$

Using $U(q, p, \theta) \equiv u(q, \theta) - p$ and assuming that the mechanism (q, p) is at least piece-wise differentiable (what could be rigorously proved), this is equivalent to:

$$\forall \theta \in \Theta, \begin{cases} \frac{dp}{d\theta}(\theta) = \frac{\partial u}{\partial q}(q(\theta), \theta) \frac{dq}{d\theta}(\theta) \\ \frac{d^2 p}{d\theta^2}(\theta) \geq \frac{\partial^2 u}{\partial q^2}(q(\theta), \theta) \left(\frac{dq}{d\theta}(\theta) \right)^2 + \frac{\partial u}{\partial q}(q(\theta), \theta) \frac{d^2 q}{d\theta^2}(\theta) \end{cases}$$

Now, if we differentiate the first-order condition and substitute it into the second-order, we get more simply:

$$\forall \theta \in \Theta, \begin{cases} \frac{dp}{d\theta}(\theta) = \frac{\partial u}{\partial q}(q(\theta), \theta) \frac{dq}{d\theta}(\theta) \\ \frac{\partial^2 u}{\partial q \partial \theta}(q(\theta), \theta) \frac{dq}{d\theta}(\theta) \geq 0 \end{cases}$$

Therefore, assuming

$$\frac{\partial^2 u}{\partial q \partial \theta}(q, \theta) > 0 \;\; \forall \theta, \forall q \tag{6}$$

a menu of contract will be incentive compatible if and only if, $\forall \theta \in \Theta$

[5] We therefore consider ex-post participation constraint. In some cases, for example when the asymmetric information regards the cost of a task for the agent, it is more realistic to assume that he discovers his type after having contracted with the principal and therefore analyze ex-ante participation constraints.

$$
\begin{cases}
\dfrac{dp}{d\theta}(\theta) = \dfrac{\partial u}{\partial q}(q(\theta),\theta)\dfrac{dq}{d\theta}(\theta) & (7) \\
\dfrac{dq}{d\theta}(\theta) \geq 0 & (8)
\end{cases}
$$

This means, in particular, that for (q,p) to be a direct truthful mechanism q (the quantity of goods offered) has to be non-decreasing with type.[6]

Condition (6) amounts here to a single-crossing condition and is referred to as the Spence-Mirrlees condition. Indeed, as we assumed $U(q,p,\theta) \equiv u(q,\theta) - p$, the slope of the indifference curves in the contract plan (q,p):

$$-\frac{\frac{\partial U}{\partial q}(q,p,\theta)}{\frac{\partial U}{\partial p}(q,p,\theta)}$$

simply writes

$$\frac{\partial u}{\partial q}(q,\theta)$$

and condition (6) states that the slope of these indifference curves increases with types for any level of q. Therefore, the indifference curves of two different types can only cross once, and two different types cannot be indifferent toward the same two contracts. From an economic point of view, this condition states that higher types are willing to pay more for an increase in quantity ($\frac{\partial}{\partial\theta}\left(\frac{\partial u}{\partial q}(q,\theta)\right) > 0$). Note here that this condition is straightforwardly verified for linear preferences $(u(q,\theta) = \theta.\mu(q))$.

Let us now turn to participation constraints. Assuming that reservation utility (*i.e.* the outside option) is independent of types[7] and normalizing it to zero, by the revelation principle, the participation constraint of type-θ agent writes:

$$v(\theta) \equiv V(\theta,\theta) = u(q(\theta),\theta) - p(\theta) \geq 0 \tag{9}$$

At the optimum, $v(\theta)$ represents the informational rent of a type-θ Agent and by (7):

$$v'(\theta) = \frac{\partial u}{\partial q}(q(\theta),\theta)\frac{dq}{d\theta}(\theta) + \frac{\partial u}{\partial \theta}(q(\theta),\theta) - \frac{dp}{d\theta}(\theta) = \frac{\partial u}{\partial \theta}(q(\theta),\theta) \geq 0 \tag{10}$$

Therefore, as in the discrete case above, the informational rent is increasing in type. Again, this informational rent can be understood as the surplus the Principal has to release to the Agent for him to reveal a higher type. By (9) and (10), one also gets that (all) the participation constraints boil down to $v(\underline{\theta}) = 0$ (as the objective of the principal is increasing in p). Thus, as above, the lowest-type Agent doesn't get any surplus (or informational rent). Then, using (10) again, we obtain $\forall\theta$

$$v(\theta) = \int_{\underline{\theta}}^{\theta} \frac{\partial u}{\partial \theta}(q(\tau),\tau)d\tau \tag{11}$$

[6]This result can be obtained using more directly the incentive constraints. For all $\theta,\theta' \in \Theta^2$, the incentive constraints are $u(q(\theta),\theta) - p(\theta) \geq u(q(\theta'),\theta) - p(\theta')$ and $u(q(\theta'),\theta') - p(\theta') \geq u(q(\theta),\theta') - p(\theta)$. Adding the two gives, $\forall\theta,\theta'$: $u(q(\theta),\theta) - u(q(\theta),\theta') \geq u(q(\theta'),\theta) - u(q(\theta'),\theta')$ *i.e*: $\int_{\theta'}^{\theta} \frac{\partial u}{\partial \theta}(q(\theta),s)ds \geq \int_{\theta'}^{\theta} \frac{\partial u}{\partial \theta}(q(\theta'),s)ds$, or: $\int_{\theta'}^{\theta}\int_{q(\theta')}^{q(\theta)} \frac{\partial^2 u}{\partial q \partial \theta}(q,s)dqds \geq 0$. Thus, if $\frac{\partial^2 u}{\partial q \partial \theta}(q,s) > 0$, $q(\theta)$ has to be non-decreasing.

[7]This assumption simplifies a lot the analysis. For an analysis of type-dependant reservation utility, see Jullien, B., "Participation constraints in adverse selection models", *Journal of Economic Theory*, 93, 1-47, 2000.

and $\forall\theta$

$$p(\theta) = u(q(\theta),\theta) - v(\theta) = u(q(\theta),\theta) - \int_{\underline{\theta}}^{\theta} \frac{\partial u}{\partial \theta}(q(\tau),\tau)d\tau$$

Now, the principal objective:

$$\int_{\underline{\theta}}^{\overline{\theta}} \left(p(\theta) - C(q(\theta))\right) f(\theta)d\theta$$

can be written as:

$$\int_{\underline{\theta}}^{\overline{\theta}} \left(u(q(\theta),\theta) - \int_{\underline{\theta}}^{\theta} \frac{\partial u}{\partial \theta}(q(\tau),\tau)d\tau - C(q(\theta)) \right) f(\theta)d\theta \tag{12}$$

and the only remaining constraint is (8): $\dfrac{dq}{d\theta}(\theta) \geq 0$.

Interestingly, (12) can be written as the first-best social surplus:

$$\int_{\underline{\theta}}^{\overline{\theta}} \left(u(q(\theta),\theta) - C(q(\theta))\right) f(\theta)d\theta$$

minus the term

$$I = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} \frac{\partial u}{\partial \theta}(q(\tau),\tau)d\tau f(\theta)d\theta \tag{13}$$

that, as above, represents the cost of incentives, that is the informational rents.
Indeed, integrating by part (13) gives:[8]

$$I = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial u}{\partial \theta}(q(\theta),\theta)[1 - F(\theta)]d\theta \tag{14}$$

Noticing that, by (10), at the optimum $\frac{\partial u}{\partial \theta}(q(\theta),\theta) = v'(\theta)$, one can see that this second term measures the (negative) effect of asymmetric information on efficiency. This terms comes from the fact that – for the contracts to be incentive compatible – the Principal has to keep informational rents increasing in type. For quantities to increase in θ, on needs to provide higher informational rent to higher θ, but this mechanism is cumulative! $(1-F(\theta))$ captures this effect: it measures the mass of agents of types $\theta' > \theta$.

Using (14), the objective of the Principal simply writes:

$$\int_{\underline{\theta}}^{\overline{\theta}} \left(u(q(\theta),\theta) - C(q(\theta)) - \frac{\partial u}{\partial \theta}(q(\theta),\theta)\frac{1}{h(\theta)} \right) f(\theta)d\theta$$

where

$$h(\theta) \equiv \frac{f(\theta)}{1 - F(\theta)}$$

[8] Letting $A = \int_{\underline{\theta}}^{\theta} \frac{\partial u}{\partial \theta}(q(\tau),\tau)d\tau$ and $B' = f(\theta)$, we have $I = \int_{\underline{\theta}}^{\overline{\theta}} AB'd\theta = [AB]_{\underline{\theta}}^{\overline{\theta}} - \int_{\underline{\theta}}^{\overline{\theta}} A'Bd\theta = \left[\int_{\underline{\theta}}^{\theta} \frac{\partial u}{\partial \theta}(q(\tau),\tau)d\tau F(\theta)\right]_{\underline{\theta}}^{\overline{\theta}} - \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial u}{\partial \theta}(q(\theta),\theta)F(\theta)d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial u}{\partial \theta}(q(\theta),\theta)\left[1 - F(\theta)\right]d\theta$

is referred to as the hazard rate.[9]

Thus, omitting the constraint (8), the optimal contract would be $q^*(\theta)$ such that, $\forall\theta$:

$$\frac{\partial u}{\partial q}(q^*(\theta),\theta) = C'(q^*(\theta)) - \frac{\partial^2 u}{\partial q \partial \theta}(q^*(\theta),\theta)\frac{1}{h(\theta)} \tag{15}$$

(it is obtained by maximizing in q the integrand of the objective at every θ). We retrieve here that asymmetric information leads to inefficient allocations as by (6), marginal utility is then lower than marginal cost, and thus every type under-consume.

If $q^*(\theta)$ coming from (15) is non-decreasing in θ, it is indeed the optimal contract. It would for example be the case when utility is linear in θ ($u(q,\theta) = \theta.\mu(q)$), provided that costs are convex and the hazard rate is non-decreasing:[10] in this case (15) writes

$$u'(q^*(\theta))\left(\theta - \frac{1}{h(\theta)}\right) = C'(q^*(\theta))$$

and differentiating with respect to θ one obtains

$$\frac{dq^*}{d\theta}(\theta) = -\frac{u'(q^*(\theta))\left(1 + \frac{h'(\theta)}{(h(\theta))^2}\right)}{u''(q^*(\theta)\left(\theta - \frac{1}{h(\theta)}\right) - C''(q^*(\theta))}$$

More generally, it could be that the solution of (15) doesn't satisfy the monotonicity condition (8): $\frac{dq}{d\theta} \geq 0$. In such cases, the constraint necessarily binds for some θs, meaning that different types are offered the same contract. We then talk about bunching. The issue is here to determine the interval(s) on which q is constant. A proper examination of these cases calls for optimal control techniques. We provide here the main intuitions on the determination of these intervals and refer the interested reader to section 2.3.3.3 of Bolton & Dewatripont (for example) for a more complete examination.

First note that, whenever the optimal contract is increasing in type it has – by optimality – to coincide with $q^*(\cdot)$ defined in (15). Now if we assume that $q^*(\cdot)$ is decreasing on the interval $[\theta_1,\theta_2]$, with $\theta_1 > \underline{\theta}$ and $\theta_2 < \overline{\theta}$, the aim is here to find the optimal interval $[\theta_0,\theta_3]$ (with $\theta_0 < \theta_1$ and $\theta_3 > \theta_2$) on which q is constant and to find the corresponding optimal level of q.[11] Defining

$$H(q(\theta),\theta) \equiv \left(u(q(\theta),\theta) - C(q(\theta)) - \frac{\partial u}{\partial \theta}(q(\theta),\theta)\frac{1}{h(\theta)}\right)$$

(that is the integrand of the Principal's objective), the problem writes:

$$\max_{\theta_0,\theta_3,q}\left\{\int_{\underline{\theta}}^{\theta_0} H(q^*(\theta),\theta)d\theta + \int_{\theta_0}^{\theta_3} H(q,\theta)d\theta + \int_{\theta_3}^{\overline{\theta}} H(q^*(\theta),\theta)d\theta\right\} \tag{16}$$

Optimization with respect to θ_0 and θ_3 gives $q = q^*(\theta_0) = q^*(\theta_3)$, that is the continuity of the

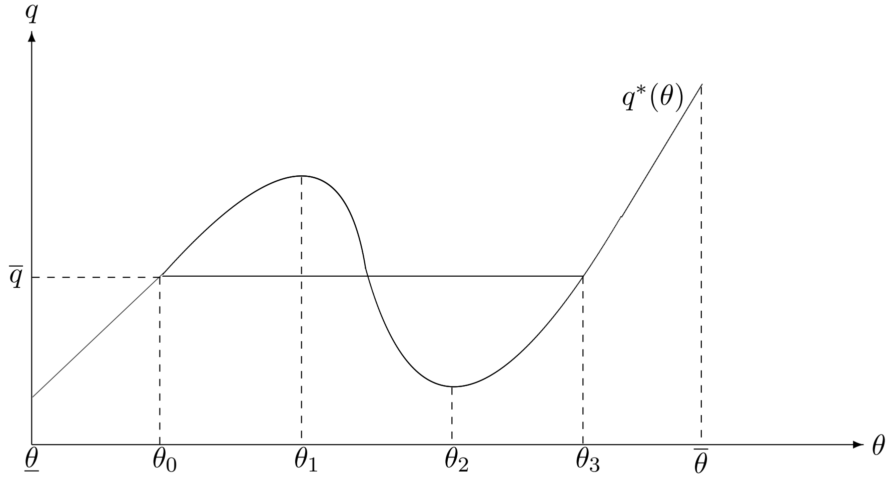
[9] $h(\theta)$ represents the probability of being of type θ conditional of being of a type non-lower that θ.

[10] This property holds for most of the classical distributions.

[11] The two simplifications are here (i) $q^*(\cdot)$ is decreasing on only one interval and (ii) bunching doesn't occur at the bottom, nor at the top of the distribution.

overall solution; and the optimal quantity $\overline{q}$ is then given by $\int_{\theta_0}^{\theta_3} \frac{\partial H}{\partial q}(\overline{q},\theta)d\theta = 0$, that is:

$$\int_{\theta_0}^{\theta_3} \left\{ \frac{\partial u}{\partial q}(\overline{q},\theta) - C'(\overline{q}) - \frac{\partial^2 u}{\partial q \partial \theta}(\overline{q},\theta)\frac{1}{h(\theta)} \right\} d\theta = 0 \tag{17}$$



2.4 Applications and extensions

Let us now study how the framework developed above can be applied to various situations. We discuss more precisely in this section applications to the credit market, natural monopolies and task delegation (that is the inner functioning of a firm).

2.4.1 Credit rationing

(see e.g. Bolton and Dewatripont section 2.2.1)

A first natural application of adverse selection concerns the credit market. Indeed, in most of the credit relationship, it is likely that the borrowers knows better the characteristics of his project (notably in terms of risk) than the lender.[12] In our framework, the borrower will then be the Agent and the lender the Principal (we will therefore somehow assume a monopolistic lender). Our aim is here to analyze the inefficiency generated by asymmetric information, and to do so a model with only two types of borrowers is enough.

Consider a risk-neutral borrower who has a project for which he needs financing. He has no collateral nor personal investment and therefore needs to borrow from the lender (let's call it the bank) the total funds for the project, that we normalize to 1. If undertaken, the project generates a return ρ_i in case of success and 0 in case of failure. We assume that projects (or borrowers) are heterogeneous with respect to their probability of success: θ_i, and their return in case of success: ρ_i. More precisely, we assume that their exists two types of projects: safe and risky, with different probability of success: respectively θ_S and θ_R, with $\theta_S > \theta_R$, but the same expected returns: $\theta_S.\rho_S = \theta_R.\rho_R \equiv \Upsilon > 1$ (leading to $\rho_R > \rho_S$).

The (monopolistic) bank offers to lend funds against repayment in case of success. We however assume that it doesn't hold enough funds to serve the entire market (in other words the market

[12]Risk assessment techniques as scoring methods aim at reducing this asymmetry of information, and in some cases – as microcredit for example – the asymmetry might even be reversed.

is characterized by excess demand): the bank can lend a total amount $K < 1$ whereas there exists a mass one of consumers. Still, denoting by α the proportion of safe borrower, we let $K > \max\{\alpha, 1-\alpha\}$, meaning that excess demand is not sufficient by itself to explain complete exclusion of one type of borrowers. We consider here cases in which the bank doesn't observe (neither ex-ante nor ex-post) the type of the borrower, meaning that it is able to observe success or failure of the project but not its return (otherwise the contract could specify different repayments for ρ_R and ρ_S).

Let us assume first that the bank only offers one contract with repayment D and highlight the issue of adverse selection. Here, the participation constraint of a borrower of type i simply writes $\rho_i \geq D$ and the bank optimally sets D either equal to ρ_S or to ρ_R. As, by assumption $\rho_R > \rho_S$, if it sets $D = \rho_R$, the bank only lends to type-R borrowers and its expected profit equals:

$$(1-\alpha)(\Upsilon - 1) \tag{18}$$

If it instead sets $D = \rho_S$, both types apply, and assuming that all borrowers have equal chance of being financed (we will discuss this assumption in the following), the expected profit of the bank is

$$K(\alpha(\Upsilon - 1) - (1-\alpha)(\theta_R.\rho_S - 1)) \tag{19}$$

In this case, the bank "looses" some money on risky borrowers ($\rho_S < \rho_R \Rightarrow \theta_R.\rho_S < \Upsilon$) to attract safe ones and use all of its funds.
The optimal strategy for the bank depends on parameter values but one can easily see that the second strategy would prevail when α is high and the difference between borrowers' types is low. In this case, credit rationing occurs: some risky borrowers don't get funded although they would be ready to accept a higher repayment rate.

To escape such adverse selection, the issue is here to find a second dimension (on top of D) on which to define the contracts and build a menu. We saw above that the probability for a borrower to be financed was key to the analysis. We will therefore analyze menus in which contracts differ both in terms of required repayment (D_i) in case of success, and of probability to get financed, that we will denote x_i.[13] The optimal menu $\{(D_S, x_S), (D_R, x_R)\}$ will then be solution of:

$$\begin{aligned}
\max_{D_S, x_S, D_R, x_R} \quad & \alpha.x_S\,(\theta_S.D_S - 1) + (1-\alpha)x_R\,(\theta_R.D_R - 1) \\
s.t. \quad & \rho_S \geq D_S && (\text{PC}_S) \\
& \rho_R \geq D_R && (\text{PC}_R) \\
& x_S.\theta_S(\rho_S - D_S) \geq x_R.\theta_S(\rho_S - D_R) && (\text{IC}_S) \\
& x_R.\theta_R(\rho_R - D_R) \geq x_S.\theta_R(\rho_R - D_S) && (\text{IC}_R) \\
& \alpha.x_S + (1-\alpha)x_R \leq K \\
& 0 \leq x_i \leq 1 \qquad \forall i = S, R
\end{aligned}$$

This program differs from the one studied in section 2.1, (i) as the Agent's type enter directly

[13] This framework can also be easily adapted to model heterogeneity in the probability of being refinanced, in the context of a repeated relationship.

the Principal's program (through θ_is) and (ii) because of the two last constraints. Still, using the same method as in section 2.1, one can easily show that the binding constraints are (PC$_S$) and (IC$_R$), leading to $D_S = \rho_S$ ("safe" borrowers don't get any surplus) and $D_R = \rho_R - \frac{x_S}{x_R}(\rho_R - \rho_S)$ ("risky" borrowers receive an informational rent). The problem then becomes:

$$\begin{aligned} \max_{x_S, x_R} \quad & \alpha . x_S \left(\Upsilon - 1\right) + (1-\alpha)\left[x_R \left(\Upsilon - 1\right) - x_S . \theta_R(\rho_R - \rho_S)\right] \\ \text{s.t.} \quad & \alpha . x_S + (1-\alpha) x_R \leq K \\ & 0 \leq x_i \leq 1 \qquad \forall i = S, R \end{aligned}$$

The monotony of the objective in x_R then gives $x_R = 1$ and either

$$x_S = \begin{cases} 0 & \text{if } \left[\alpha \left(\Upsilon - 1\right) - (1-\alpha).\theta_R(\rho_R - \rho_S)\right] < 0 \\ \left[K - (1-\alpha)\right]/\alpha & \text{otherwise} \end{cases}$$

In the first case, "risky" borrowers are offered the first-best contract $x_R = 1$ and $D_R = \rho_R$, at the cost – for the bank – of not using all its funds; whereas in the second, the bank has to offer them an informational rent $D_R < \rho_R$ in order to also finance "safe" borrowers. This last case occurs when safe borrowers are numerous enough.

Note here that credit rationing doesn't occur at the optimum, as the only borrowers who don't get financing, *i.e.* type-S borrowers, are indifferent between taking up the credit or not. However, this result heavily relies on the assumption that $\theta_S.\rho_S = \theta_R.\rho_R > 1$, that is on the fact that both types are attractive for the bank. If instead $\theta_S.\rho_S > 1$ but $\theta_R.\rho_R < 1$ the bank would like to get rid of risky borrowers but cannot. Depending on parameters, this would lead either to financial collapse (no one gets credit) or to the above mentioned pooling equilibrium with $D = \rho_S$ for all borrowers (in which safe borrowers then subsidize risky ones). This last case will be preferred by the bank provided it then makes profit, that is if

$$\left[\alpha.\theta_S + (1-\alpha)\theta_R\right]\rho_S > 1$$

2.4.2 Regulation of natural monopolies

(see e.g. Bolton and Dewatripont section 2.2.4)

Another important application of adverse selection is the regulation of natural monopolies. The issue is here for a regulator (the Principal) – who doesn't observe the cost structure of a natural monopoly (the Agent) – to protect consumers welfare, and make the monopoly charge the competitive price. Rather than assuming that costs are unobservable, we assume that costs are observable and contractible, but are composed of two individually unobservable components: "intrinsic productivity" θ and "effort to cut costs" e: $c = \theta - e$. We assume that θ is exogenous and can take two values: θ_L and θ_H, with $\theta_H > \theta_L$; whereas $e > 0$ is endogeneous, chosen optimally by the monopolist and has quadratic costs: $\psi(e) = e^2/2$. We then have a simple case with both hidden information and hidden action.

To provide the monopoly an incentive to decrease its cost (though e) although charging a price equal to its cost, the regulator pays subsidies s and we assume that it aims at minimizing the total expenses of producing the good: $P = c + s$.[14] The profit of the monopolist then equals $P - c - \psi(e) = s - \psi(e)$.

[14] One can understand P as the total price of the good assuming that the subsidy is financed through taxes.

If the regulator would observe the type of the monopolist (θ), through the observation of total costs, it would be able to infer the effort (e). It would then be able to condition s on e and the first-best contract would be the solution of the program:

$$\begin{aligned} \min_{s,e} \quad & \theta - e + s \\ \text{s.t.} \quad & s - e^2/2 \geq 0 \end{aligned}$$

leading to $e^* = 1$ and $s^* = 0.5$. At the first-best, both types of firm receive the same subsidy and exert the same effort, leading to the same profit but with different cost (and different prices).

Now, if the regulator doesn't observe θ, but only its distribution (as usual $\alpha = \mathbb{P}(\theta = \theta_L)$), it has to propose a menu $\{(s_L, c_L), (s_H, c_H)\}$, using the fact that it observes overall cost. In this case, if a type-i firm chooses the contact (s_j, c_j), $j \neq i$, it has to provide a level of effort $\tilde{e}_i$ such that its total cost equals to one of a type-j firm: $\theta_i - \tilde{e}_i = c_j = \theta_j - e_j$, *i.e.* $\tilde{e}_i = e_j + (\theta_i - \theta_j)$. The optimal menu will then be the solution of the problem (as θ_is are exogenous, choosing c_i amounts to choosing $e_i = \theta_i - c_i$):

$$\begin{aligned} \min_{s_L, e_L, s_H, e_H} \quad & \alpha(\theta_L - e_L + s_L) + (1-\alpha)(\theta_H - e_H + s_H) \\ \text{s.t.} \quad & s_L - e_L^2/2 \geq 0 && (\text{PC}_L) \\ & s_H - e_H^2/2 \geq 0 && (\text{PC}_H) \\ & s_L - e_L^2/2 \geq s_H - \left[e_H + (\theta_L - \theta_H)\right]^2/2 && (\text{IC}_L) \\ & s_H - e_H^2/2 \geq s_L - \left[e_L + (\theta_H - \theta_L)\right]^2/2 && (\text{IC}_H) \end{aligned}$$

We assume that $e_H > \theta_H - \theta_L$, *i.e.* $c_H < \theta_L$ so that (IC_L) is indeed needed. Otherwise a type-L firm could not mimic the costs structure of a type-H.

The issue comes here from the fact that an efficient (type-L) firm has an incentive to mimic inefficient (type-H) ones and their costs structure, as it will result in lower effort. Therefore, and as before, the relevant (and binding) constraints are (PC_H) and (IC_L). This yields – using the fact that θ_is are exogeneous – to

$$\min_{e_L, e_H} \alpha \left(-e_L + e_L^2/2 + e_H^2/2 - \left[e_H + (\theta_L - \theta_H)\right]^2/2 + \right) + (1-\alpha)(-e_H + e_H^2/2)$$

whose solution are:

$$e_L = 1 \quad \text{and} \quad e_H = 1 - \tfrac{\alpha}{1-\alpha}(\theta_H - \theta_L)$$

We thus find again the now classical "efficiency at the top" and informational rent (which is decreasing with the effort provided by inefficient firms). At the optimum, the subsidy paid to an inefficient firm only covers its optimal cost-reducing effort (leading to zero-profit), whereas an efficient firm makes positive profit but provide first-best effort.

2.4.3 Delegation and audit

(see e.g. Laffont and Martimort section 3.6)

We have assumed up to now that incentives were the only mean for the Principal to "detect" the Agent type, and informational rent the only way for her to prevent non-truthful reports.

In many real-life examples however, the Principal can have access to audit technologies that allows detecting the agent type ex-post. These technologies are however costly and are often used in addition to incentivized contracts. Let us analyze such situations in the context of task delegation.

Consider a firm (the Principal) who has to delegate to a worker (the Agent) the production of a good against a wage w. The principal earns a surplus $S(q)$ (with $S'(\cdot) > 0$, $S''(\cdot) < 0$ and $S(0) = 0$) for the production of q units of the good, and faces two types of workers that differs in terms of production efficiency (*i.e.* in terms of production costs). Formally, the type of the Agent: θ represents his marginal cost of production:

$$C(q, \theta) = \theta.q$$

and can take two values θ_L and $\theta_H > \theta_L$.

Ignoring first the possibility of auditing the agent, the problem is symmetric to the one studied in section 2.1. If the contracting variable are q and w, the first-best contract (with θ observable) solves

$$\begin{aligned} \max_{q,w} \quad & S(q) - w \\ \text{s.t.} \quad & w - \theta.q \geq 0 \end{aligned}$$

Thus, $S'(q_L^*) = \theta_L$ and $S'(q_H^*) = \theta_H$ (quantities are socially optimal) with $w_L^* = q_L^* \theta_L$ and $w_H^* = q_H^* \theta_H$ (such that delegation is costless for the principal).
Now, when θ is unobservable and defining $\alpha = \mathbb{P}(\theta = \theta_L)$, the problem is:

$$\begin{aligned} \max_{q_L, w_L, q_H, w_H} \quad & \alpha(S(q_L) - w_L) + (1-\alpha)(S(q_H) - w_H) \\ \text{s.t.} \quad & w_L - \theta_L.q_L \geq 0 \\ & w_H - \theta_H.q_H \geq 0 \\ & w_L - \theta_L q_L \geq w_H - \theta_L q_H \\ & w_H - \theta_H q_H \geq w_L - \theta_H q_L \end{aligned}$$

Giving, using exactly the same techniques as above:

- efficiency at the top: $S'(\tilde{q}_L) = \theta_L$
- downward distortion of output for the "bad" type: $S'(\tilde{q}_H) = \theta_H + \frac{\alpha}{1-\alpha}(\theta_H - \theta_L)$
- no surplus for the "bad" type: $\tilde{w}_H = \theta_H \tilde{q}_H$
- an informational rent for the efficient worker: $\tilde{w}_L = \theta_L \tilde{q}_L + (\theta_H - \theta_L)\tilde{q}_H$

Now assume that the Principal owns an audit technology that allows her to discover the true type of the Agent with probability λ at a cost $c(\lambda)$, with $c'(\cdot) > 0$, $c''(\cdot) > 0$ and $c(0) = 0$. For both incentives and audit to be used at the optimum, we also assume Inada conditions: $c'(0) = 0$ and $\lim_{\lambda \to 1} c'(\lambda) = +\infty$. The possibility of auditing enlarges greatly the contracting possibilities and, assuming away the possibility of rewarding truthful revelation, we will consider contracts of types (w, q, λ, Λ) where λ is the probability of audit and Λ represents the punishment in case of non-truthful report. A menu $\{(w_L, q_L, \lambda_L, \Lambda_L), (w_H, q_H, \lambda_H, \Lambda_H)\}$ is then incentive-compatible if and only if

$$w_L - \theta_L q_L \geq w_H - \theta_L q_H - \lambda_H \Lambda_L \tag{IC$_L$}$$
$$w_H - \theta_H q_H \geq w_L - \theta_H q_L - \lambda_L \Lambda_H \tag{IC$_H$}$$

(λ_H is indeed the probability of auditing an agent announcing type θ_H and Λ_L the punishment for a type θ_L doing a non-truthful report).

One can easily see from the above that audit softens the incentive issue, as soon as probabilities and punishment are strictly positive. By the Revelation Principle, we can then focus on the direct mechanisms satisfying (IC$_L$), (IC$_H$) and the usual participation constraints:

$$w_L - \theta_L q_L \geq 0 \tag{PC$_L$}$$
$$w_H - \theta_H q_H \geq 0 \tag{PC$_H$}$$

The Revelation Principle has important implications here. Indeed, the mechanism being truthful, the audit will never detect any lie. Still, the principal has to commit on an (ex-post inefficient) auditing strategy to soften the incentive issue. Moreover, focusing on trustful reports, the objective of the principal writes

$$\alpha(S(q_L) - w_L - c(\lambda_L)) + (1-\alpha)(S(q_H) - w_H - c(\lambda_H)) \tag{20}$$

and it is worth noticing that punishments are absents from it.

For legal or liability reasons we assume that punishment cannot be infinite and that it is constrained either endogenously on endogenously. In the case of endogenous punishments, we will constrain punishments not to exceed the surplus gained by the lying agent. Then:

$$\Lambda_L \leq w_H - \theta_L q_H \tag{LL$_L$}$$
$$\Lambda_H \leq w_L - \theta_H q_L \tag{LL$_H$}$$

On the contrary, in the case of exogenous punishments, the bound on punishments in independent on the contract and the same for both types:

$$\Lambda_L \leq l \tag{LP$_L$}$$
$$\Lambda_H \leq l \tag{LP$_H$}$$

The Principal problem is then to maximize (20) over $\{(w_L, q_L, \lambda_L, \Lambda_L), (w_H, q_H, \lambda_H, \Lambda_H)\}$ subject to constraints (PC$_L$), (PC$_H$), (IC$_L$), (IC$_H$) and either $\{$(LL$_L$), (LL$_H$)$\}$ or $\{$(LP$_L$), (LP$_H$)$\}$.

We consider here cases in which the only relevant incentive constraint is (IC$_L$) and the only relevant participation constraint is (PC$_H$).[15] In these cases, the constraint (LL$_L$) (or (LP$_L$)) necessarily binds at the optimum: raising the punishment, the Principal soften the incentive constraint at no cost. This mechanism is referred to as the "Maximal Punishment Principle". On the contrary, as the incentive constraint of a type-θ_H Agent is always slack at the optimum, the Principal never audits a Agent announcing a "good" type θ_L: $\lambda_L = 0$ (and Λ_H is irrelevant).

[15] Contrarily to the cases studied above, this is not always the case due to punishments.

The constraints such boils down to:

$$\begin{aligned} w_H &= \theta_H q_H \\ \Lambda_L &= (\theta_H - \theta_L) q_H \\ w_L &= \theta_L q_L + (1 - \lambda_H)(\theta_H - \theta_L) q_H \end{aligned}$$

in the case of endogenous punishment. And the problem is then:

$$\max_{q_H, q_L, \lambda_H} \quad \alpha(S(q_L) - \theta_L q_L - (1 - \lambda_H)(\theta_H - \theta_L) q_H) + (1 - \alpha)(S(q_H) - \theta_H q_H - c(\lambda_H))$$

This gives:

- $S'(q_L^a) = \theta_L$ that is $q_L^a = q_L^*$: efficiency at the top
- $S'(q_H^a) = \theta_H + \frac{\alpha}{1-\alpha}(1 - \lambda_H^a)(\theta_H - \theta_L)$, and $\tilde{q}_H < \overline{q^a} < \overline{q^*}$: the effect of audit on incentives allows reducing the distortions caused by asymmetric information (an increases in λ_H has the same effect on w_L – *i.e.* on informational rent – than a decrease in q_H.)
- $c'(\lambda_H^a) = \frac{\alpha}{1-\alpha}(\theta_H - \theta_L) q_H^a$: the optimal audit probability trade-off its cost its benefit in terms of reduction of the informational rent.

Now, in the case of exogenous punishment, the binding constraints write:

$$\begin{aligned} w_H &= \theta_H q_H \\ \Lambda_L &= l \\ w_L &= \theta_L q_L + (\theta_H - \theta_L) q_H - \lambda_H l \end{aligned}$$

and the optimal menu solves:

$$\max_{q_H, q_L, \lambda_H} \quad \alpha(S(q_L) - \theta_L q_L - (\theta_H - \theta_L) q_H + \lambda_H l) + (1 - \alpha)(S(q_H) - \theta_H q_H - c(\lambda_H))$$

This gives:

- $S'(q_L^a) = \theta_L$ that is $q_L^a = q_L^*$: efficiency at the top
- $S'(q_H^a) = \theta_H + \frac{\alpha}{1-\alpha}(\theta_H - \theta_L)$, and $q_H^a = \tilde{q}_H$: audit is useful in reducing transfers but doesn't impact productive choices.
- $c'(\lambda_H^a) = \frac{\alpha}{1-\alpha} l$: again, the optimal audit probability trade-off its cost its benefit in terms of reduction of the informational rent.

One can moreover notice that our analysis is – in this case – only valid for low enough values of l. We have indeed assumed that (PC_L) was slack, what will only by the case (see the expression of w_L) if $\lambda_H l < (\theta_H - \theta_L) q_H$.

2.5 Signaling models

We have analyzed up to now situations in which the asymmetry of information was to the benefit of the Agent. Let us now analyze what happens when the Principal (*i.e.* the party with all the bargaining power) is the one having private information. The inefficiencies generated by this kind of asymmetry has been highlighted by Akerlof (1970)[16] in his analysis of the market for second-hand cars. After having presented this basic problem, we will show that allowing the informed party to send a "signal" prior to the transaction might help reducing inefficiencies.

2.5.1 The basic problem: market for lemons

(see e.g. Salanié section 4.1)

Consider the market for second-hand cars and assume that their exists two types of cars in the market: good cars (in proportion α) and bad cars, called "lemons". The type of a car determines both the floor price of the seller (that can be understood as her cost): respectively g and b; and the cap price of the buyer (his willingness to pay): respectively G and B. We naturally assume $B < G$ and $b < g$, and consider cases in which there are opportunities to trade: $G > g$ and $B > b$. Then, assuming that the seller have all the bargaining power (she is then the Principal)[17], under symmetric information goods cars would be sold at price G, and lemons at price B.[18]

Now assume that the type of the car is only known to the seller. If she has no mean to convince the buyer of the quality of the car (we will analyze this possibility in the next section), the seller cannot price-discriminate between car types. Still, she is not willing to sell a good car at a price lower than g. On the other side – without any information – a buyer is not willing to pay more than $\hat{\alpha}.G + (1 - \hat{\alpha})B$ for any car, where $\hat{\alpha}$ represent the effective proportion of good car in the market (or at least his belief about it). The buyer having all the bargaining power, the price will indeed be set to $\hat{\alpha}.G + (1 - \hat{\alpha})B$ and either:

- $\alpha.G+(1-\alpha)B \geq g$ and both type of cars are traded ($\hat{\alpha} = \alpha$) at price $p = \alpha.G+(1-\alpha)B$,[19] or
- $\alpha.G + (1 - \alpha)B < g$ and only lemons are traded ($\hat{\alpha} = 0$) at price $p = B$.

In the second case, observing a price lower than g, the buyer infers that the car is a lemon and therefore is only willing to pay a price B. In these situations, *i.e.* when α is low, asymmetric information causes important inefficiency: although there were gains to do so ($G > g$), good cars are not traded at equilibrium. We therefore have adverse selection also in cases in which the Principal is the party with private information. In the next section, we analyze to what extent introducing the possibility for the Principal to take an action before contracting (for example here by getting a certification by an independent mechanic) can reduce inefficiencies.

[16] Akerlof, G., "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism", *The Quarterly Journal of Economics*, 84 (3), 488-500, 1970.
[17] This occurs for example if the number of cars is finite but the number of potential buyers (the demand) is infinite.
[18] Both types would also be traded if no one had information on the type of the car. All cars would then be sold at price $\alpha.G + (1 - \alpha)B$.
[19] Note that price then coincides with the case in which no party observes the type of the car.

2.5.2 Education as a Signal

(see e.g. Bolton and Dewatripont section 3.1)

To analyze the effect of a signaling stage taking place before the contracting stage, let us analyze the case of the labor market and consider education as a signal of productivity (in the spirit of Spence, 1973)[20]. We assume more precisely that workers differ in their intrinsic marginal productivity ω_i and can signal their ability through education,[21] and consider two types of worker: $i = \{H, L\}$, with $\omega_H > \omega_L$ and $\alpha = \mathbb{P}(H)$ (*i.e.* the proportion of type-H agents). We will confer to education a signaling effect by assuming that (i) the marginal cost of education is lower for high-productivity workers: it will cost $c(y) = \theta_i.y$ for a type-i worker to acquire y years of education (above the legal requirement) with $\theta_H < \theta_L$, (ii) productivity is unobservable and (iii) years of education are observable. We focus on the signaling effect of education and don't consider here the positive effect education can have on productivity. (Therefore, if ω_i would be observable, every worker would choose $y = 0$). We consider the following functioning of the labor market:

- firms are willing to hire any worker at any wage lower than her expected productivity
- workers are willing to work at any positive wage
- workers have all the bargaining power (*e.g.* firms compete *à la* Bertrand for each worker)

We are then is a Principal-Agent setting in which the Principal has private information. What we add with respect to the previous section is a first-stage in which the Principal choose a level of education, *i.e.* a signal. At this stage, a worker of type i optimally chooses

$$y_i^* \in \arg\max_y w(y) - \theta_i y \tag{21}$$

where $w(y)$ represent her expected wage if she gets y years of education. As in the previous section, this wage will depend on the beliefs of the Agent about the productivity of a worker with education y:

$$w(y) = \hat{\alpha}(H \mid y)\omega_H + (1 - \hat{\alpha}(H \mid y))\omega_L \tag{22}$$

where $\hat{\alpha}(H \mid y)$ represents the belief of a firm about the probability that a worker with education y is of type H. As we saw above, this belief should vary with the strategy of the Principal, and if one wants it to be consistent with the effective probabilities, one should focus on the concept of Perfect Bayesian Equilibrium (PBE).

We consider here the set of Perfect Bayesian Equilibria in pure strategies, and refer the reader to the section 3.1 of Bolton and Dewatripont for an analysis of mixed strategies. A PBE is then a set of strategy y_i chosen by the Principal and of beliefs $\hat{\alpha}(\omega_H \mid y_i)$ of the Agent, that verify (21), (22) and are consistent with effective probabilities (*i.e.* satisfy Bayes' rule):

$$\hat{\alpha}(H \mid y_i) = \mathbb{P}(H \mid y_i) = \frac{\mathbb{P}(y_i \mid H)\alpha}{\mathbb{P}(y_i \mid H)\alpha + \mathbb{P}(y_i \mid L)(1-\alpha)} \tag{23}$$

(Note here that beliefs about levels of education other than those chosen at equilibrium, *i.e.* $\hat{\alpha}(H \mid y)$ for $y \notin \{y_L, y_H\}$, are not constrained).

[20] Spence, M., "Job Market Signaling", *The Quarterly Journal of Economics*, 87(3), 355-374, 1973.

[21] We consider here cases in which signals are costly. For a discussion of costless signals – and the corresponding cheap-talk model – the reader is referred to the section 4.3 of Salanié's book.

In pure strategies, with only two types, a PBE can either be a separating (different types choose different strategies) or a pooling equilibrium (both types choose the same strategy).[22]

In our setting, a separating equilibrium (y_L, y_H) will be a PBE, if:

- $y_L \neq y_H$
- $\hat{\alpha}(H \mid y_H) = 1$ and $\hat{\alpha}(H \mid y_L) = 0$, such that $w(y_H) = \omega_H$ and $w(y_L) = \omega_L$
- as education is the useless for a type-L Principal (they cannot get less than ω_L): $y_L = 0$
- a type-L Principal doesn't want to mimic a type-H Principal (otherwise beliefs would not be consistent)

$$\begin{aligned} & w(y_L) - \theta_L.y_L \geq w(y_H) - \theta_L.y_H \\ \Leftrightarrow \quad & y_H \geq \frac{\omega_H - \omega_L}{\theta_L} \end{aligned}$$

- a type-H Principal doesn't want to mimic a type-L Principal

$$\begin{aligned} & w(y_H) - \theta_H.y_H \geq w(y_L) - \theta_H.y_L \\ \Leftrightarrow \quad & y_H \leq \frac{\omega_H - \omega_L}{\theta_H} \end{aligned}$$

As out-equilibrium beliefs are unrestricted ($\mathbb{P}(H \mid y)$ is undefined for $y \notin \{y_L, y_H\}$) – without equilibrium refinement – one cannot further determine y_H (see equation (21)), and we end up with a continuum of separating PBE:

$$\left\{(y_L, y_H) : y_L = 0 \text{ and } y_H \in \left[\frac{\omega_H - \omega_L}{\theta_L}, \frac{\omega_H - \omega_L}{\theta_H}\right]\right\}$$

Similarly, in our setting, a pooling equilibrium will be a PBE if

- $y_H = y_L = y_P$
- $\hat{\alpha}(H \mid y_P) = \alpha$ such that $w(y_P) = \alpha.\omega_H + (1-\alpha).\omega_L$
- every type of Principal is willing to participate: $w(y_P) - \theta_i.y_P \geq 0$
- every type of Principal prefers $(y_P, w(y_P))$ to a situation where she doesn't invest in education and gets the lowest wage ω_L: $w(y_P) - \theta_i \geq \omega_L$.
  As $\theta_L > \theta_H$ and $\omega_L > 0$ the two former points boil down to:

$$y_P \leq \frac{\alpha.(\omega_H - \omega_L)}{\theta_L}$$

Again – without equilibrium refinement – there then exists a continuum of pooling equilibria:

$$\left\{(y_L, y_H) : y_L = y_H = y_P \text{ and } y_P \in \left[0, \frac{\alpha.(\omega_H - \omega_L)}{\theta_L}\right]\right\}$$

[22] In mixed strategies another type of equilibrium, called semi-separating, arises (see *e.g.* section 3.1 of Bolton and Dewatripont).

Note here that all workers prefer a situation in which education is absent (they then get a wage $\alpha.\omega_H + (1-\alpha).\omega_L$) at no cost to any of these pooling equilibrium. A situation where education (*i.e.* signaling) is impossible even Pareto-dominates all the pooling PBE as in both cases the firm makes null expected profit.

We still end up with a multiplicity of equilibrium (as often with PBE): a continuum of separating PBE and a continuum of pooling PBE. Still – as for pooling equilibria – one separating equilibrium Pareto-dominates the others: the one with $y_L = 0$ and $y_H = \frac{\omega_H - \omega_L}{\theta_L}$. This PBE (often referred to as the "least-cost" separating equilibrium) is the one reached by most of the refinement studied in the literature, be it by constraining the set of out-of-equilibrium beliefs (for example the one proposed by Cho and Kreps, 1987,[23] see *e.g.* Bolton and Dewatripont section 3.1.1.1) or by changing the timing of the game and defining wages contingent on education (Maskin and Tirole, 1992,[24] see *e.g.* Bolton and Dewatripont section 3.1.1.2). In this case, the equilibrium shares a lot of properties with the optimal contract under screening (section 2.1 to 2.4): only one incentive constraint is binding (the one of low-productivity agent) and only one type gets the efficient allocation at the optimum ($y_L = 0$). Still, this result relies on specific refinements of PBE, and in a more general setting, equilibria are multiple and might be driven by specific social norms or conventions. In other words, the resulting allocation is prone to self-fulfilling prophecies.

2.5.3 Application to corporate finance

(see e.g. Freixas and Rochet[25] section 3.2)

As already discussed, the credit market is one natural application of hidden information models. We have analyzed in section 2.4.1 the case of a monopolistic lender trying to screen among borrowers. Let us now turn to the case of a competitive capital market in which project-owners try to signal the quality of their project. In the following, we will show that a signal can consist in investing ones own fund in the project, *i.e* in retaining some of the risk. For this signal to be costly we will assume that project-owners (that we will call entrepreneurs) are risk-averse, and more precisely that their utility functions satisfies CARA: $u(x) = -e^{-A.x}$, $A > 0$.

Consider an entrepreneur endowed with a risky project of size 1 that generates a random return $\rho(\theta) = 1 + r(\theta)$, with $r(\theta) \sim \mathcal{N}(\theta, \sigma^2)$. Although she holds enough wealth to finance this project (we denote by $W > 1$ her initial wealth), because of risk-aversion, she is willing to sell her projects on the capital market. Assuming a competitive capital market, would θ be observable, each project would be sold at price $\mathbb{E}(r(\theta)) = \theta$, such that the entrepreneur would reach (sure) utility $u(W + \theta)$.

Now, if θ is only known to the entrepreneurs, as in the above cases, the investors cannot discriminate among projects and offer a common price E of equity for all the project. This price would be acceptable for a type-θ entrepreneur provided it gives her a higher expected utility that financing the project with her own funds, *i.e.* if and only if:

$$u(W + E) \geq \mathbb{E}(u(W + r(\theta))$$

[23] Cho, I.-K. and Kreps, D., "Signaling Games and Stable Equilibria", *The Quarterly Journal of Economics*, 102(2), pp. 179-221, 1987.

[24] Maskin E. and Tirole, J., "The Principal-Agent Relationship with an Informed Principal, II: Common Values", *Econometrica*, 60 (1), pp. 1-42, 1992.

[25] Freixas, X. and Rochet, J.-C., *Microeconomics of Banking*, MIT Press.

or, using the fact that $u(\cdot)$ is CARA and $r(\cdot)$ is Gaussian[26], if and only if:

$$u(W+E) \geq u\left(W+\theta-\frac{1}{2}A\sigma^2\right).$$

Offering to buy equity at price E would therefore attract only entrepreneurs of type

$$\theta \leq E+\frac{1}{2}A\sigma^2$$

Thus at equilibrium – when the capital market is competitive – equity is bought at a price such that:

$$E=\mathbb{E}\left(\theta \mid \theta \leq E+\frac{1}{2}A\sigma^2\right)$$

In the simple case with two type of entrepreneurs: H and L, with $\theta_H > \theta_L$ and $\alpha = \mathbb{P}(H)$, this equilibrium will be inefficient (and adverse selection will occur) if type-H entrepreneurs are not willing to accept the equilibrium price that would prevail if both type would participate, that is if:

$$\begin{aligned} & \theta_H > \mathbb{E}(\theta)+\frac{1}{2}A\sigma^2 \\ \Leftrightarrow \quad & A < 2\frac{(1-\alpha)}{\sigma^2}(\theta_H-\theta_L) \end{aligned}$$

It will then be the case if risk-aversion is too low to compensate the adverse selection effect. In this case type-H entrepreneurs will prefer to fully self-finance their project.

Now assume that entrepreneurs can mix between self-financing and equity-financing. Because of risk aversion, they always would prefer to rely as much as possible of equity. Still partial self-finance (which is costly) might be used by "good-quality" entrepreneurs to signal their type. Let us focus on the case with two types of entrepreneurs and the "least-cost" separating equilibrium. Following the above discussion, this equilibrium will be characterized by:

- full equity financing of L-type entrepreneurs at price θ_L
- partial equity financing of H-type entrepreneurs at price θ_H,
- a level of self-financing by H-type binding the incentive constraint of L-type entrepreneurs:

$$\begin{aligned} \kappa: \quad & u(W+\theta_L)=\mathbb{E}(W+\kappa r(\theta_L)+(1-\kappa)\theta_H) \\ \Leftrightarrow \quad & u(W+\theta_L)=u\left(W+\kappa\theta_L+(1-\kappa)\theta_H-\frac{1}{2}A\sigma^2\kappa^2\right) \\ \Leftrightarrow \quad & \frac{\kappa^2}{1-\kappa}=\frac{2(\theta_H-\theta_L)}{A\sigma^2} \end{aligned} \tag{24}$$

The equilibrium level of self-financing by "good-quality" entrepreneurs is then increasing with the extent of adverse selection (*i.e.* the difference between types) and decreasing with risk and risk aversion. At equilibrium, H-type borrowers gets an expected utility equal to:

$$\mathbb{E}u\left(W+\kappa r(\theta_H)+(1-\kappa)\theta_H\right)=u\left(W+\theta_H-\frac{1}{2}A\sigma^2\kappa^2\right)$$

[26]When $u(\cdot)$ is CARA and $X \sim \mathcal{N}(\mu,\sigma^2)$ basic calculations give $\mathbb{E}(u(X))=u(\mu-1/2A\sigma^2)$

and the inefficiency caused by the asymmetry is then measured by the term $\frac{1}{2}A\sigma^2\kappa^2$ with κ defined in (24).

2.6 Dynamic aspects: renegotiation and commitment

(see e.g. Bolton and Dewatripont section 9.1)

Let us consider now contexts in which the relationship between the informed and the uninformed parties is repeated. When the type of the informed party doesn't evolve through time, revealing its type impacts future contracts thereby modifying incentives.

We focus here on cases in which the Agent is the informed party. The issue is then twofold. Once types are revealed, (i) the Principal does not need to provide a "good"-type Agent with an information rent anymore, (ii) as mimicking is then no more possible, she can provide a "bad"-type Agent with his efficient allocation. This deeply modifies the incentives of a "good"-type Agent to reveal his type and therefore the optimal contracts.

A useful example that highlights these two issues is the repeated regulation of natural monopoly.[27] Consider the situation of section 2.4.2 repeated during two periods, with a common discount factor δ for the regulator and both types of firms.

If the regulator is able to commit from the first period on contracts for two periods, its problem consists of choosing $s_{L1}, e_{L1}, s_{H1}, e_{H1}, s_{L2}, e_{L2}, s_{H2}, e_{H2}$ solutions of:

$$\begin{aligned}
\min \quad & \alpha\left[s_{L1} - e_{L1} + \delta(s_{L2} - e_{L2})\right] + (1-\alpha)\left[s_{H1} - e_{H1} + \delta(s_{H2} - e_{H2})\right] \\
\text{s.t.} \quad & s_{L1} - \frac{e_{L1}^2}{2} + \delta\left[s_{L2} - \frac{e_{L2}^2}{2}\right] \geq 0 && (\text{PC}_L) \\
& s_{H1} - \frac{e_{H1}^2}{2} + \delta\left[s_{H2} - \frac{e_{H2}^2}{2}\right] \geq 0 && (\text{PC}_H) \\
& s_{L1} - \frac{e_{L1}^2}{2} + \delta\left[s_{L2} - \frac{e_{L2}^2}{2}\right] \geq s_{H1} - \frac{(e_{H1}-\Delta\theta)^2}{2} + \delta\left[s_{H2} - \frac{(e_{H2}-\Delta\theta)^2}{2}\right] && (\text{IC}_L) \\
& s_{H1} - \frac{e_{H1}^2}{2} + \delta\left[s_{H2} - \frac{e_{H2}^2}{2}\right] \geq s_{L1} - \frac{(e_{L1}+\Delta\theta)^2}{2} + \delta\left[s_{L2} - \frac{(e_{L2}+\Delta\theta)^2}{2}\right] && (\text{IC}_H)
\end{aligned}$$

where $\Delta\theta \equiv \theta_H - \theta_L$.

In our simple case with a quadratic cost of effort, it can be shown (see *e.g.* Laffont and Tirole, 1993[28]) that the optimal strategy is to offer during both period the optimal one-period contract found in section 2.4.2. Indeed, in this case, efficient firms provide optimal effort and, specifying time specific level of effort for inefficient firms ($e_{H1} \neq e_{H2}$) doesn't soften neither the participation constraint of inefficient firms (PC$_H$) (because of the convexity of costs), nor the incentive constraint of efficient firms (IC$_L$). We then have: $e_{L1}^* = e_{L2}^* = 1$, $e_{H1}^* = e_{H2}^* = 1 - \frac{\alpha}{1-\alpha}\Delta\theta$ and s_{H1}^*, s_{H2}^*, s_{L1}^*, s_{L2}^* such that (PC$_H$) and (IC$_L$) binds.

[27] Another classical example is the case of a monopolist selling a durable good (see *e.g.* Bolton and Dewatripont section 9.1.1). The issue is then for the monopolist to prevent high-valuation consumers from deferring consumption after anticipating that prices would decrease in the future to attract consumers with lower valuation.

[28] Laffont, J.-J. and Tirole, J., *A Theory of Incentives in Procurement and Regulation*, MIT Press, 1993.

Now, as discussed above, once types are revealed, the Principal may want to use the information she gathers in period one to reach a more efficient allocation in period 2. Assuming that the first period contract becomes the default contract (we discuss the case in which the contracts are independent next), this comes to looking for possible Pareto-improving renegotiation. The L-type providing efficient effort and receiving an information rent in the default contract, the only Pareto-improvement would come from an increase in the effort of H-type firms. As usual, one can deal with this issue by analyzing renegotiation-proof contracts, *i.e.* contracts in which the allocation is efficient for both types. As (PC_H) and (IC_L) will again be the binding constrains, the problem is then:

$$
\begin{aligned}
\min \quad & \alpha\left[s_{L1} - e_{L1} + \delta(s_{L2} - e_{L2})\right] + (1-\alpha)\left[s_{H1} - e_{H1} + \delta(s_{H2} - e_{H2})\right] \\
\text{s.t.} \quad & s_{H1} - \frac{e_{H1}^2}{2} + \delta\left[s_{H2} - \frac{e_{H2}^2}{2}\right] = 0 \\
& s_{L1} - \frac{e_{L1}^2}{2} + \delta\left[s_{L2} - \frac{e_{L2}^2}{2}\right] = s_{H1} - \frac{(e_{H1} - \Delta\theta)^2}{2} + \delta\left[s_{H2} - \frac{(e_{H2} - \Delta\theta)^2}{2}\right] \\
& e_{L2} = e_{H2} = 1
\end{aligned}
$$

giving again $e_{L1} = 1$ and $e_{H1} = 1 - \frac{\alpha}{1-\alpha}$, but with a higher informational rent for L-type firms (due to the increase in e_{H2}, by (IC_L)). As the Principal will always be interested ex-post in increasing the level of effort exerted by H-type firms, and L-type ones anticipate this, the possibility of renegotiation leads to an increase in the informational rent of efficient firms. Note here that another renegotiation-proof allocation consists in a pooling contract in period 1 (with $e_L = 1$ and $e_H = 1 - \Delta\theta$, such that $c_L = c_H$) and the optimal one-period separating contract in period 2.

Now – if contracts are fully independent between the two periods – once the Agent-type is revealed, the Principal doesn't need to provide informational rent anymore. Then, the possible contracts are: (i) the pooling contract defined above and (ii) a separating contract in which types are revealed in period-one and thus both types of firms extract no surplus from period-2. With respect to the previous case, rents can then only be given in period-1, and participation constraints are now period-specific. This modifies deeply the incentive constraints that become:

$$
s_{L1} - \frac{e_{L1}^2}{2} \geq s_{H1} - \frac{(e_{H1} - \Delta\theta)^2}{2} + \delta\left[\frac{e_{H2}^2}{2} - \frac{(e_{H2} - \Delta\theta)^2}{2}\right]
$$

for the efficient type, and

$$
s_{H1} - \frac{e_{H1}^2}{2} \geq s_{L1} - \frac{(e_{L1} + \Delta\theta)^2}{2}
$$

for the inefficient one. This last constraint reflects the fact that after mimicking a type L in period one, a type H firm is not interested in renewing the contract, as it would give it a negative surplus (indeed $s_{L2} - e_{L2}^2/2 = 0 \Rightarrow s_{L2} - (e_{L2} + \Delta\theta)^2/2 < 0$). In this case, both incentive constraints are likely to bind at the optimum, ending up in the pooling contract defined above as the unique equilibrium. Repetition then jeopardizes type-revelation and therefore conveys inefficiency.

3 Hidden action: The issue of moral hazard (D. Henriet)

3.1 Reminder on the basic model

3.2 Applications and extensions

3.3 Moral hazard in teams

3.4 Dynamic aspects: carrier concerns

4 The limits of the theory of incentives (D. Henriet)

4.1 Countervailing incentives

4.2 Behavioral aspects: intrinsic motivation