Cash reserve policy, regulation and credibility in insurance

Renaud Bourlès[†] and Dominique Henriet [‡]

ABSTRACT

The aim of this paper is to analyze, with a very simple model, the different market failures that motivate cash and default regulation in insurance. Proponents of deregulation argue that the main market failure concerns information of customers about the insurer's ability to meet his commitment. According to them, this can be efficiently solved by the obligation to disclose information on solvency margins. Adding to the purpose the relationship between the insurer and her security holders (that is the issuance and dividend policy) we show that this disclosure policy is not sufficient to restore efficiency because of limited commitment on recapitalization.

Subject headings: insurance, cash reserve, regulation, recapitalization

1. Introduction

Is cash regulation necessary, and, if so, which kind is the most accurate? In all countries that have insurance markets, regulation of insurance companies exists. The main motivation of such a regulation seems to be the protection of insurance buyers against the risk of insolvency of their insurers. This regulation generally takes the form of "technical" or "mathematical" reserve that insurers should at least carry in sufficiently liquid and riskless assets. These regulatory reserve are expressed as ratios of premium income and claims expenses. We want to focus in this paper on the necessity and the economic motivations of such regulation rules.

[†]Ecole Centrale Marseille and GREQAM-IDEP, renaud.bourles@centrale-marseille.fr

[‡]Ecole Centrale Marseille and GREQAM-IDEP, dominique.henriet@centrale-marseille.fr

The authors are indebted, with the usual disclaimers, to Mohamed Belhaj and Jean-Charles Rochet for their precious comments and suggestions. We are also grateful to all the participants who attended our seminar in Toulousefor helpful discussion. Part of this work has been done when the first author was lecturer at University Toulouse 1.

The first rationale for cash regulation seems to be the potential asymmetry of information between insurers and policyholders. The buyer of insurance pays a premium against the promise that she will receive a payment if the specified random events occur. If the insurer does not hold enough reserve to fulfill this promise, the consumer is being cheated ex ante. This potential failure, itself, undermines the confidence on which the market is based. If the policyholders does not observe this default risk, the insurance market faces a typical "lemons problem": uncertainty about product quality – here solvency – may drive high-quality firms out of the market. Regulation is then intended to make sure that only "good" firms (with low risk of insolvency) are in the market.

Proponents of deregulation however argue that cash requirement is not an appropriate tool to mitigate this adverse selection problem. As the problem arises because policyholders are not conveniently informed about the risk of insolvency, "information disclosure" policy, that is the public provision of information about insurers' risk of insolvency, is sufficient to solve the adverse selection issue. Private incentives are then sufficiently high to induce companies to hold enough liquid capital to optimally reduce the risk of insolvency.

Such a reasoning is however silent on other key actors of insolvency: shareholders or debt owners. Indeed, for a company to become insolvent, not only cash has to be insufficient to meet claims but it also has to be suboptimal to recapitalize (or impossible to issue debt). In the present paper we therefore want to focus on possible other reasons of cash regulation, beside the relation "insurer/policyholder" evoked above. Another bilateral relationship – between the insurer and her security holders – indeed seems to be of interest. It is now well established that agency problems may arise from the asymmetry of information between managers and shareholders (see La Porta et al. 2000 for an overview of these theories and for empirical tests that support them). For example, managers can invest in inefficient projects that generate private benefits for them to the detriment of shareholders. Such an issue would therefore give security holders an incentive not to leave cash in the insurance company. The problem becomes clearer in presence of frictional capital market, if we assume that issuing new debt is costly. An interesting trade-off then arise between agency cost and recapitalization cost.

To study these mechanisms we build a simple discrete-time dynamic model of insurance and analyze the behavior of security holders. On the one hand, when the company is solvent, that is when assets are sufficient to meet claims, the security holders can either take dividend or issue new shares (or debt). On the other hand, if claims are too large for the current cash reserve to cover it, the security holders choose whether to recapitalize the insurance company or to default.

In this context we study three settings. Firstly, as benchmark, we analyze a situation where the insurer is able to commit on a recapitalization policy. We moreover assume in this case that policyholders have perfect information on the reserve of their insurer and on its dividend and issuance policy. In such a context, we show that because on the effect it has on the set of premia acceptable for policyholders, the insurance company – that is its shareholders – is better off committing on always recapitalizing (even when the needed capital exceeds the future value of dividend payments). The intuition behind this result is that, because of policyholders' risk-aversion, it is always profitable to increase the range of indemnified losses: the cost of such a policy is lower than the incremental income it allows to collect (through premia). It is however difficult for shareholders to make such a issuance policy credible ex-ante, as they would ex-post be better off in the case of default if the amount of cash needed to indemnify losses is greater than the expected value of future dividends.

We therefore also examine the case where shareholders are unable to commit on a recapitalization policy. In such situations, default is endogenous and arises when the amount of recapitalization needed to keep on operating is greater than the value of the insurance company. We then analyze two different cases depending on the information policyholders have on the cash reserve of their insurer: asymmetric information and information disclosure. In the case where policyholders have no information of these reserve, they form expectations on it, which lead to an expectation on the reservation premium (that depends on the expected default probability of their insurer). When the capital market is frictionless, that is when issuing new shares is costless, the optimal strategy consists in taking dividend as long as it is possible – because of agency cost – and to recapitalize each time it is needed (provided the future value of the company is larger than the invested capital). However, if issuing new debt is costly, it can be optimal to leave some cash in the company. The optimal strategy is then a barrier strategy: take dividend above a bottom limit, neither take dividend nor issue new debt if the ex-post (cash) reserve is positive but below the limit and issue new shares in order to meet claims when the current reserve is insufficient.

When the information on cash reserve is disclosed, the insurer can use its reserve as a signal of low default probability. In this context we show that – even in the case of frictionless capital market – it can optimal be optimal to leave cash in the company. By increasing its reserve, the firm can increase the premia, without loosing any policyholders. Then, by increasing the premia acceptable by the policyholders – through the use of strictly positive reserve – disclosure of information leads to a higher value for the insurer than under asymmetric information. Therefore, disclosure of information reduces the probability of default. However, default still endogenously occurs as recapitalization is credible only when the amount of needed cash is below the value of the firm. Disclosure of information on the cash reserve of the insurer is thus not enough to restore the first best, that is to guarantee that no default occurs.

An efficient regulation would therefore consist in making the commitment to always recapitalize credible (what would be a good thing for the company) or at least in guaranteeing that the company would always hold enough asset to continue operating. It therefore appears that setting up a guarantee fund can be a more efficient regulation than cash requirement.

We briefly discuss the relationship of this paper with the literature. Our work fits in the literature on cash reserve and solvency in insurance. Initiated by Borch (1981) in a model where shareholders can only invest in capital during the first period, this literature has then developed in analyzing the optimal dynamic choice of capital. Munch and Smallwood (1981) and Finsinger and Pauly (1984) for example analyze capital choices in a situation where the demand for insurance is elastic with respect to default risk. Both papers however assume that shareholders cannot recapitalize after claims are realized.

In a more recent paper, Rees, Gravelle and Wambach (1999) study a situation in which policyholders are fully informed of the default probability of their insurer. They show that, whereas an unconstrained insurer will optimally choose a corner solution (either zero or maximum), once the policyholders are informed about the probability of not being indemnified, the insurer's expected value is higher if it holds the maximum amount of capital. Rees, Gravelle and Wambach (1999) however ignore the possibility of recapitalization when claims exceed assets. They indeed assume that contracts are not fully honored in these cases. Under this assumption insurers can commit on a default probability through their cash reserve. Being informed of the amount of cash their insurer holds, individuals can infer the probability of not being paid. Competition in insurance market then lead the companies to raise the maximum amount of cash. However, as we introduce recapitalization – that is the possibility to reinject cash when claims exceed assets – this mechanism no longer holds. Insurers then cannot commit on a default probability as they cannot commit on the behavior of their security holders. This creates a motive for an internal solution for cash reserve and therefore a room for cash regulation if this solution is suboptimal.

Blazenko, Parker and Pavlov (2007, 2008) analyze the concept of "economic ruin" by modeling a situation where new shares can be issued in case of deficit. They however assume an exogenous dividend policy in the sense that a fixed return (the risk-free interest rate) is paid to shareholders whenever cash reserve are positive, and that insurer can continue operating with negative capital (debt). We however want to focus here on optimal (and therefore endogenous) issuance and dividend policy and we assume that shareholders has to recapitalize a company with negative capital if they want the company not to default. Finally, we want to focus on a different regulation scheme than Blazenko, Parker and Pavlov

(2007, 2008). They indeed consider a regulation that requires an immediate cash contribution to offset a capital deficit when we analyze cash requirement and the setting up of a guarantee fund.

Our work is also related to the actuarial analysis of dividend strategies. Starting with de Finetti (1957), an extensive literature studies the dividend and issuance strategy of an insurance company (see Avanzi 2009 for a survey). If the barrier strategy we find in the case of asymmetric information is pretty common in this literature (see for example Gerber 1979 or Gerber and Shiu 2004), it is generally found for a given distribution of claims whereas we do not impose any restriction on the distribution of claims[1]. Moreover, we add to this literature the modeling of the behavior of policyholders. Whereas, all these papers deal with exogenous premia, we endogenize in this work the premia charged by the insurer through the participation choice of policyholders.

The sequel of the paper is organized as follows. In the next section (Section 2) we first present the features of the general discrete-time dynamic model. In section 3 we present the simple (one period) static case. This allows two understand three cases of information structure and commitment settings: (i) the first best, with full information on reserve and full commitment on recapitalization, (ii) the lemon effect, when there is no possible commitment on default policy because of unobservability of the cash reserve and (iii) information disclosure on cash reserve which can then be used as a commitment tool. Section 4 is devoted to the results of the general dynamic model. We study the implication on the optimal issuance and dividend policy of different information and commitment settings. Such an approach allows us to capture the need for cash regulation and to analyze in Section 5 the efficiency of two forms of regulation: cash requirement and the setting up of a guarantee fund. Conclusions and directions for future research are eventually provided in Section 6.

[1] Note here that we nonetheless impose in our discrete time model that all claims occur at the same time.

2. The model

We consider a discrete-time dynamic model where an insurance company offers, at each period, insurance contracts to n identical policyholders. Each policyholder incurs a per period loss $\widetilde{x}_i$ on her income w. We denote by $\widetilde{x}_t$ the total claim at date t. We note f the distribution of $\widetilde{x}_t$, F its CDF and e its mean. Policyholders value wealth through a von Neumann, increasing and strictly concave, utility function u.

The timing inside a given period, from date t to $t+1$ is the following.

- At the beginning of each period the insurance company, if active, holds some assets m_t (cash reserve or liquid assets).
- He proposes a (full) insurance contract (premium π_t) to the n potential customers, who can either accept or refuse, comparing it to some outside option.
- so that (if the contract is accepted) total assets amounts to $A_t = n\pi_t + m_t$.
- For sake of simplicity, we assume that claims occur at the beginning of the period. Two cases are then possible.
  - Either, in a first case, the total claim x_t is lower than total assets $A_t = n\pi_t + m_t$. Shareholders of the company can then recapitalize or take dividends for the following period. Let $k_t(x_t)$ the amount of recapitalization (if negative, $-k_t(x_t)$ is the amount of dividend payments). In the following period, the insurance company then begins with a new cash reserve that amounts to: $m_{t+1} = \rho(A_t - x_t + k_t(x_t))$, where ρ is the return on the cash left in the company.
  - Or, in a second case, the total claim x_t is larger than $A_t = n\pi_t + m_t$ and the company is potentially insolvent. The security holders can either refuse to keep on operating – in that case insured are not fully indemnified and A_t is simply equally shared among them – or subscribe to an issue of new securities (shares or debt[2]) to meet the claims. We suppose that there is a potential cost of issuance: 1 dollar of fresh cash in the company costs $\gamma \geq 1$ to the security holders. This can be, for example, explained by transaction costs (see Gomes 2001 for a justification and an evaluation of these costs). Notice here that there we do not allow for negative

[2] In the following model we focus on share issuance. Most of our results seems to remain with debt issuance but such a modeling would add interest paiement and debt maturity to the purpose.

balance sheet[3]. If $\widetilde{x}$ is larger than A_t the company must either stop or issue new shares.

Shareholders discount future with a discount rate δ. We moreover assume $\delta \leq 1/\rho$. This means that shareholders prefer to consume immediately rather than to invest in the risky technology: the insurance company. This may for example reflect agency problems (for a justification of this assumption see La Porta et al. 2000 or Rochet and Villeneuve 2005). Put another way, this means that the rate of return required by the shareholders is greater than the internal rate of return. There is hence no direct incentive to leave cash in the company. Note here, that most of the paper on this topic (see Avanzi for a survey) assume that internal cash has a return equal to one. This might reflect the fact that reserve has to be liquid. Our results remain for $\rho = 1$.

In the following, we describe alternative information and commitment frameworks. To begin with, it is useful to study the one period (static) case and distinguish three cases: (i) the full commitment and full information case (first best), (ii) the case of asymmetric information on reserve, and (iii) the case of information disclosure on reserve.

3. The static model

Consider first the static case where the relation between the insurer and the insured only lasts one period. This allows to apprehend the main mechanism of our model and to better describe the different setting we analyze. In this case, dividend policy is pretty simple: the entire profit (in any) is distributed as dividend at the end of the period.

At the beginning of the period the insurer offers a full insurance contract with premium π, holds a cash reserve m and defines a default policy summarized by the set I of total loss x that will be indemnified. The consumers then accept (or refuse) the contract comparing it to an outside option that provides a given level of expected utility. At the end of the period, risk occurs, and contracts are enforced (according to the information and the commitment settings). In this static model, we moreover suppose for the sake of simplicity that $\gamma = 1$ (i.e. that issuance is costless).

[3]Note here that permanent debt would anyway be suboptimal as it would induce new commitment issues.

3.1. The static, full commitment, perfect information benchmark

Suppose first that the insurer can commit on a default policy and that policyholders can observe their insurer's reserve. The default policy is summarized by the set I of values of the total loss x that are indemnified. If x does not belong to I, the insurer defaults. The program of the insurer therefore consists in setting the premium π, the reserve m and the range I of indemnified losses in order to maximize her expected profit under the constraint that policyholders accept the contract:

$$\max_{\pi,I,m\geq 0}\int_I \delta\rho\left[(m+n\pi)-x\right]f(x)dx-m$$
$$\int_I u(w-\pi)f(x)dx+\int_{I^C}u\left(w+\frac{m-x}{n}\right)f(x)dx\geq \underline{u}$$

Result 1 *If the company can commit on a reserve policy, it commit on no default:* $I=[0,+\infty)$*. Therefore, cash reserve can be maintained to zero:* $m=0$*, and the premium* π *is defined by* $u(w-\underline{\pi})=\underline{u}$.

The optimal profit of the insurer therefore rights $\delta\rho\left(n\underline{\pi}-e\right)$. As soon as $n\underline{\pi}\geq e$, that is if the contract is actuarially profitable (what would be assume throughout the paper[4]), in this first best framework the insurer never defaults and never holds initial assets.

It therefore appears that, if the insurer can commit on a cash reserve policy, it is of his interest to commit to never default. This commitment allows him to set the highest acceptable premium. Moreover the insurer does not need to put any initial cash as it can commit to put it ex post.

3.2. The static case with no commitment

3.2.1. Imperfect information: the lemon effect

Suppose now that there is no mean for the insurer to commit on a default policy, or that such a commitment is not credible. The acceptable premium for policyholders will then depend on their expectations. These expectations will be highly influenced by the observed or suspected reserve of the insurer.

[4]Therefore, it would never be on the interest of the insurer to default. The only basis for default will then be the lack of credibility.

If cash reserve is private information for the insurer, policyholders can only have beliefs I^e and m^e on the default and cash reserve policies. This leads to a reservation premium π^e above which the contract is not accepted. The program of the insurer therefore writes

$$\max_{I,m\geq 0} \int_I \delta\rho \left[(m + n\pi^e) - x\right] f(x)dx - m$$

which leads to the following result

Result 2 *If the insurer is unable to commit on a default policy and if its cash reserve is private information, initial cash reserve is optimally set to zero:* $m = 0$ *and shareholders never recapitalize the insurance company:* $I = [0, n\pi^e)$.

As the insurer can not commit a default policy, and as it only operates one period, the only credible policy is to default as soon as claims exceed cash reserve. As moreover, initial cash reserve is not observable it can not be used as a signal of solvency. Therefore m is set to zero and the insurer default as soon as claims exceed collected premia: $I = [0, n\pi^e)$.

The rational expectation equilibrium is hence the value of π^{e*} solution of:

$$u(w - \pi)F(n\pi) + \int_{n\pi}^{+\infty} u\left(w - \frac{x}{n}\right) f(x)dx = \underline{u}$$

And the optimal profit writes:

$$\int_{-\infty}^{n\pi^{e*}} \delta\rho \left[n\pi^{e*} - x\right] f(x)dx$$

which is smaller than $\delta\rho\left(n\underline{\pi} - e\right)$.

We face here the typical (market for) lemon effect: uncertainty about product quality – here solvency – drive high-quality firms out of the market. We therefore end up with insurers with null initial cash reserve.

Moreover, it may be the case that this lemon effect leads to the disappearance of the market, if the premium compatible with rational expectation is too low for the insurer to make a positive expected profit. Let us therefore analyze now the case where cash reserve are perfectly observed by policyholders and can therefore be a signal of high-quality, that is of low probability of default.

3.2.2. The static second best: cash reserve are disclosed (and perfectly observed)

In what we call the second best framework – where there is information disclosure on the cash reserve – the insurer uses its cash level as a commitment signal. As we are still in a static framework (the insurer close after the end of the period), the only credible policy is to default as soon as claims exceed cash reserve: $I = [0, m + n\pi)$. Note here that default is still endogenous, as m (and π) are set optimally by the firm. The program of the insurer now writes:

$$\max_{\pi, m \geq 0} \int_{-\infty}^{m+n\pi} \delta\rho \left[(m + n\pi) - x \right] f(x)dx - m$$
$$U(m, \pi) = u(w - \pi)F(m + n\pi) + \int_{m+n\pi}^{+\infty} u\left(w + \frac{m - x}{n} \right) f(x)dx \geq \underline{u}$$

Let $\pi_{\underline{u}}(m)$ be the higher acceptable premium that can be set by an insurer holding an amount m of cash ($\pi_{\underline{u}}(m) \equiv \pi / U(m, \pi) = \underline{u}$). We know from section 3.1. that $\lim\limits_{m \to \infty} \pi_{\underline{u}}(m) = \underline{\pi}$. Moreover, it can be easily shown that $\pi_{\underline{u}}(m)$ is increasing with m, $\pi'_{\underline{u}}(m)$ going to zero as m goes to infinity. The problem therefore amounts to maximizing:

$$\max_{m \geq 0} \int_{-\infty}^{m+n\pi_{\underline{u}}(m)} \delta\rho \left[(m + n\pi_{\underline{u}}(m)) - x \right] f(x)dx - m$$

Taking the derivative gives :

$$\delta\rho \left[1 + \pi'_{\underline{u}}(m) \right] F(m + n\pi_{\underline{u}}(m)) - 1$$

If $\delta\rho < 1$ (that is if this if shareholders prefer to consume immediately rather than leaving cash in the company), this derivative is negative for sufficiently large values of m. Moreover, for $m = 0$ it equals $\delta\rho \left[1 + \pi'_{\underline{u}}(0) \right] F(n\pi^{e*}) - 1$ which is positive if $\delta\rho$ is not too small. Then cash reserve can be profitably used as a signalling device to make the default policy credible. This may be summarized in the next result

Result 3 *If $\left[1 + \pi'_{\underline{u}}(0) \right] F(n\pi^{e*}) > \frac{1}{\delta\rho}$, the information disclosure of cash reserve involves some positive initial reserve: m^{SB}. As a result, the range of indemnified losses writes: $I = [0, m^{SB} + n\pi_{\underline{u}}(m^{SB}))$*

Confronted with Result 2 this means that the information disclosure on cash reserve reduces the probability of default of the insurer. As it also increases the premium, it turns out that disclosure of information enhances the value of the company. However, since default still occurs with some positive probability, the value achieved is smaller that the one obtained in the full commitment case.

If this static framework allows to capture some of the effects at stake, it is silent on a key feature of the model: issuance policy. Indeed, as the insurer stop operating at the end of the period, their is now reason (in the no commitment case) for the insurer to issue new shares. This won't be the case in the dynamic setting, when issuing new shares can allow the company to continue operating.

4. The dynamic case

Now turn to the dynamic model. At each period, the insurer has to decide whether to keep on operating or to default. As in the static case the default and cash policy will crucially depend on the information structure. We first examine the frictionless full commitment case. We then assume that commitment is impossible and examine two cases: asymmetric information on cash reserve and information disclosure.

4.1. The frictionless, full commitment benchmark

As in the static case, let us first examine a setting where the insurer can (ex ante, before knowing claims) commit on his default policy. This corresponds to set the range I of losses that are indemnified, or equivalently, the range I^C of losses that provoke default.

As before, increasing the range of indemnified losses allows to increase the premium acceptable by the policyholders. Provided that there is no cost to recapitalize and because of policyholders' risk-aversion, such a commitment is beneficial to the insurer.

Let $V_{FC}(m_0)$ be the value of the firm that holds initial cash reserve m_0, and let $k(x)$ be the amount of cash injected in the company when total claims are equal to x. $k(.)$ therefore represents the issuance and dividend policy: it amount to issuance if $k(.)$ is positive, and to dividend payment if $k(.)$ is negative.

We have:

$$\begin{aligned} V_{FC}(m_0) &= \max_{I,k(.),\pi} \int_I -k(x)f(x)dx + \delta \int_I V_{FC}(\rho\,(n\pi + m_0 - x + k(x)))f(x)dx \\ \text{s.t.} & \quad \int_I u(w-\pi)f(x)dx + \int_{I^C} u\left(w + \frac{m_0 - x}{n}\right) f(x)dx \geq \underline{u} \end{aligned} \tag{1}$$

As we assume that the company can not hold permanent debt (what would anyway be suboptimal), the cash reserve must be always positive, and:

$$\begin{aligned} V_{FC}(m_0) &= \max_{I,m_1(.)\geq 0,\pi} \int_I (m_0 + n\pi - x) f(x)dx + \int_I \left(\delta V_{FC}(m_1(x)) - \frac{1}{\rho} m_1(x) \right) f(x)dx \\ \text{s.t.} & \quad \int_I u(w-\pi) f(x)dx + \int_{I^C} u\left(w + \frac{m_0 - x}{n}\right) f(x)dx \geq \underline{u} \end{aligned} \tag{2}$$

where $m_1(x)$ represents the level of cash reserve decided for the next period (in the case where shareholders choose to keep on operating).

Let us now set $m_{FC} \equiv \arg\max \delta V_{FC}(m) - \frac{1}{\rho} m$ the optimal amount of cash reserve and $\delta W_{FC} \equiv \max \delta V_{FC}(m) - \frac{1}{\rho} m$ the optimal value of the insurer.

Noticing $\mu(I)$ the measure of the set I, we then have:

$$\begin{aligned} V_{FC}(m_0) &= \max_{I,\pi} \int_I (m_0 + n\pi - x) f(x)dx + \delta W_{FC} \mu(I) \\ \text{s.t.} & \quad \int_I u(w-\pi) f(x)dx + \int_{I^C} u\left(w + \frac{m - x}{n}\right) f(x)dx \geq \underline{u} \end{aligned} \tag{3}$$

It is now quite easy to derive the following proposition, consistent with our previous result:

Proposition 1 *If the insurer can commit on a default policy, in the case on costless recapitalization, it optimally commit to never defaulting.*

Proof. see appendix ■

This result comes from the behavior of policyholders. As policyholders are risk averse whereas shareholders are assumed to be risk neutral, it is always profitable for the insurer to increase the range of indemnified losses. The cost (in terms of cash) of such a policy is always lower than the incremental income it allows to collect through premia. This simplifies the participation constraint and we have:

$$\begin{aligned} V_{FC}(m) &= m + n\underline{\pi} - e + \delta W_{FC} \\ \text{with} &: \quad u(w - \underline{\pi}) = \underline{u} \end{aligned} \tag{4}$$

It follows that $m_{FC} \equiv \arg\max \left(\delta V^{FC}(m) - \frac{1}{\rho}m\right) = 0$ as soon as $\delta\rho \leq 1$

The optimal cash reserve is therefore zero. As there is no cost of recapitalization, shareholders prefer not to leave cash in the company and to issue new shares when claims exceed premia.

Proposition 2 *When there is full commitment, the optimal cash reserve is zero. Therefore, the optimal strategy consists in taking dividend as long as it is possible and to issue new shares when premia are not sufficient to pay claims.*

4.2. No commitment

In this section we now assume that there is no way for the insurer to commit on a default policy (or equivalently that such commitments are not credible). In this framework, as in the static case, we examine two cases. In the first one the level of cash reserve is not observed by the policyholders whereas in the second one we assume perfect information disclosure on cash reserve.

Because commitment is impossible, the insurer will optimally default if the amount needed to continue operating (that is to reset the cash reserve to zero) is larger than the present value of the firm. This amount obviously depends on the initial level of reserve. When this initial reserve is not observed, the behavior of the policyholders (i.e. the maximal acceptable premium) relies on beliefs. On the contrary, when it is observed, this behavior relies on the observed level of cash.

4.2.1. Asymmetric information : the lemon effect

First assume that cash reserve is unobservable and that policyholders have a belief m_e on the cash reserve, and a belief I_e on the default policy. This leads to a maximal acceptable premium π_e solution of:

$$\int_{I_e} u(w-\pi)f(x)dx + \int_{I_e^C} u\left(w + \frac{m_e - x}{n}\right) f(x)dx = \underline{u}$$

Given this belief, an insurer with initial cash m_0 maximizes the present value of the firm:

$$V_{NC}(m_0) = \max_{I,k(.)}\int_I - k(x)f(x)dx + \delta\int_I V_{NC}(\rho\,(n\pi_e + m_0 - x + k(x)))f(x)dx$$

that is:

$$V_{NC}(m_0) = \max_{I,m_1(.)\geq 0}\int_I (m_0 + n\pi_e - x)\,f(x)dx + \int_I \left(\delta V_{NC}(m_1(x)) - \frac{1}{\rho}m_1(x)\right) f(x)dx \tag{5}$$

where $m_1(x)$ represents the reserve chosen by shareholders for the next period. As we do not allow the company to operate with negative cash reserve (short term debt), $m_1(x)$ has to be positive.

Setting again $m_{NC} \equiv \arg\max_{m\geq 0}(\delta V_{NC}(m) - \frac{1}{\rho}m)$ the optimal cash reserve and $\delta W_{NC} \equiv \max_{m\geq 0}(\delta V_{NC}(m) - \frac{1}{\rho}m)$ the optimal value of the insurance company, equation (5) becomes :

$$V_{NC}(m_0) = \max_I \left(\int_I (\delta W_{NC} + m_0 + n\pi_e - x)\,f(x)dx\right) \tag{6}$$

By the envelop theorem we then have, noticing I_{NC} the optimal range of operating: $V'_{NC}(m_0) = \mu(I_{NC})$. This implies, as $\delta\rho < 1$, that $\delta V_{NC}(m) - \frac{1}{\rho}m$ is a strictly decreasing function of m, which in turn implies that $m_{NC} = 0$.

It is therefore optimal not to leave cash reserve in the company ($m_{NC} = 0$).

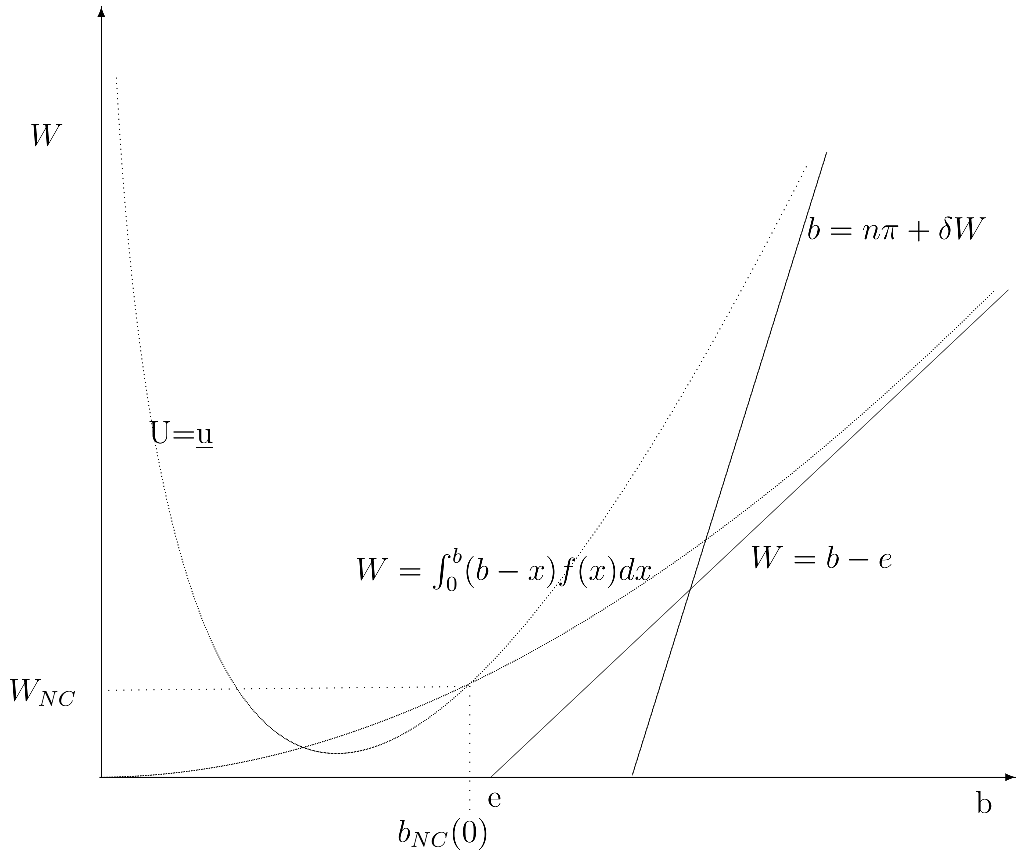
We can now deduce the optimal range I_{NC}. Noticing that is optimal to keep on operating until the integrand of (6) is positive, we have $I_{NC} = (0, b_{NC}(m_0))$, where $b_{NC}(m_0) = m_0 + n\pi_e + \delta W_{NC}$ is the default threshold. We can then conclude that $b_{NC}(0)$ and W_{NC} are solutions of the following system of equations.

$$\begin{cases} W = \int_0^b (b-x)f(x)dx \\ b = n\pi_e + \delta W \end{cases} \tag{7}$$

Then, in order to be an equilibrium, this solution must be such that the implied default policy ($b_{NC}(0)$) and cash reserve (i.e zero) lead the insured to the acceptable premium she expected π_e. In other words, $b_{NC}(0), W_{NC}, \pi_e$ must be solutions of (7) and :

$$u(w-\pi)F(b) + \int_b^{\infty} u\left(w - \frac{x}{n}\right) f(x)dx = \underline{u} \tag{8}$$

On the following picture we represent in the plane (b, W) the curves $W = \int_0^b (b-x) f(x) dx$ and $u\left(w - \frac{b}{n} + \frac{\delta W}{n}\right) F(b) + \displaystyle\int_b^\infty u\left(w - \frac{x}{n}\right) f(x) dx = \underline{u}$ (with their asymptote). The highest intersection between these two curves, if any, is the point $b_{NC}(0), W_{NC}$.



Proposition 3 *When the insurer cannot commit on a default policy and when cash reserve are private information, it is optimal for shareholders not to leave any cash in the company, that is to take dividend as long as it is possible. However, contrarily to the full commitment case, default optimally occurs when claims exceed the value of the firm, that is:* $I = [0, n\pi_e + \delta W_{NC})$ *where* π_e *and* W_{NC} *are solution of (7) and (8)*

The above result must be contrasted with the ones obtained by Rees, Gravelle and Wambach (1999). In their model default is exogenous: if cash reserve is at least enough to meet claims, the insurer remains in business and receives a continuation value that represents the expected present value of being in the insurance business at the end of the first period. If claims costs turn out to be greater than reserve, the insurer pays out his assets and defaults on the remaining claims, losing the right to the continuation value. In such a framework, they find that it can be optimal for the insurer to put enough initial cash to avoid default, provided that insurance claims distributions belong to the class of "increasing failure rate" distributions on a bounded support. This result seems to be questionable since, in particular, there is no reason to assume that cash or assets must be put ex-ante and that ex-post recapitalization is impossible. This feature can, by the way, lead to accumulate ex ante a huge (and potentially infinite) level of capital up to the maximal value of total claims. It is as if there was an infinite cost of recapitalization and a zero cost of initial capital.

We obtain here a more nuanced result: default is endogenous and optimally decided when new cash needed is too large compared with the expected returns. This leads to a policy where permanent cash is useless. The only reason for permanent cash to be useful would be the case where recapitalization is costly. Let us now examine this case.

Positive cost of recapitalization In this paragraph we assume that capital market imperfections make issuing of new shares (or new debt) costly: $\gamma > 1$. This may reflect, for example, the existence of transaction costs. In particular, when cash reserve becomes negative security holders have to choose between issuing new (costly) shares and to stop operating. The important consequence is that this cost creates an ex ante incentive to some precautionary policy which takes the form of cash reserve. Intuitively, when γ is low, reserve can be maintained to zero. But when γ, the cost of issuance, becomes larger, it turns to be optimal to hold some permanent strictly positive cash reserve.

Suppose that shareholders can recapitalize when needed, and inject $k(x)$ in the firm at a cost $\gamma > 1$. When k is negative, that is when shareholders take dividends, there is no opportunity cost. The present value of the firm is then :

$$V_{NC}(m_0) = \max_{I,k(.)} \int_I - \max(k(x), \gamma k(x)) f(x)dx + \delta \int_I V_{NC}(\rho (n\pi_e + m_0 - x + k(x))) f(x)dx$$

Which gives, noticing $m_1(x)$, the level of cash of the next period :

$$V_{NC}(m_0) = \max_{I,m_1(.)\geq 0} \int_I \varphi \left(m_0 + n\pi_e - x - \frac{1}{\rho} m_1(x) \right) f(x)dx + \int_I \left(\delta V_{NC}(m_1(x)) \right) f(x)dx$$

where $\varphi(m) = \min(m, \gamma m)$.

Let us now set

- $\overline{m_\gamma} \equiv \arg\max_{m\geq 0} \left(\delta V_{NC}(m) - \frac{1}{\rho} m \right)$ the optimal reserve after a dividend payment,
- $\underline{m_\gamma} \equiv \arg\max_{m\geq 0} \left(\delta V_{NC}(m) - \frac{\gamma}{\rho} m \right)$ the optimal reserve after the issuance of new shares,
- $\delta \overline{W}_\gamma \equiv \max \left(\delta V_{NC}(m) - \frac{1}{\rho} m \right)$ the optimal value after a dividend payment, and
- $\delta \underline{W}_\gamma \equiv \max \left(\delta V_{NC}(m) - \frac{\gamma}{\rho} m \right)$ the optimal value after the issuance of new shares.

Such a writing allows us to state the following lemma

Lemma 1 *:* $0 \leq \underline{m_\gamma} \leq \overline{m_\gamma}$ *and* $\underline{W}_\gamma \leq \overline{W}_\gamma$

Proof. see appendix ■

This gives rise to following proposition which describes the optimal strategy and the corresponding value of the firm.

Proposition 4 *The optimal cash policy is given by two optimal thresholds* $a(m_0) \leq m_0 + n\pi_e \leq b(m_0)$ *such that :*

- *if* $x \leq a(m_0)$, $m_1(x) = \overline{m_\gamma}$*: the insurer give dividends above* $\overline{m_\gamma}$
- *if* $a(m_0) \leq x \leq m_0 + n\pi_e, m_1(x) = \rho (m_0 + n\pi_e - x)$*: the insurer neither issues share nor gives dividend*

- *if* $m_0 + n\pi_e \leq x \leq b(m_0)$, $m_1(x) = \underline{m_\gamma} = 0$*: the insurer issues shares up to* $\underline{m_\gamma} = 0$
- *if* $x \geq b(m_0)$ *the company defaults.*

where $a(m_0)$, $b(m_0)$, *and* $V(m_0)$, $\overline{W}_\gamma$, $\overline{m_\gamma}$ *are solutions of :*

$$\begin{aligned}
V(m_0) &= \max_b \int_0^{a(m_0)} \left[\delta \overline{W}_\gamma + (m_0 + n\pi_e - x)\right] f(x)dx \\
&+ \int_{a(m_0)}^{m_0+n\pi_e} \delta V(\rho\,(m_0 + n\pi_e - x)) f(x)dx \\
&+ \int_{m_0+n\pi_e}^{b} \left[\delta V(0) + \gamma\,(m_0 + n\pi_e - x)\right] f(x)dx \\
a(m_0) &= m_0 + n\pi_e - \frac{\overline{m_\gamma}}{\rho} \\
\delta \overline{W}_\gamma &= \max \delta V(m) - \frac{m}{\rho} \\
\overline{m_\gamma} &= \arg\max \delta V(m) - \frac{m}{\rho}
\end{aligned}$$

Proof. see appendix ■

In this context, when issuing debt or shares is necessary, the shareholders just put enough to meet claims: $\underline{m_\gamma} = 0$. When the profit is large, shareholders take away dividends and leave some "precautionary reserve" $\overline{m_\gamma} \geq 0$. Intuitively, this capital reserve is larger as γ is large. Conversely, when γ is sufficiently low, this level can be maintained to 0. Indeed, the derivative of V w.r.t. m for $m = 0$ can be computed (thanks to the envelop theorem) :

$$\begin{aligned}
V'(0) &= F\,(a(0)) + \int_{a(0)}^{n\pi_e} \delta\rho V'(\rho\,(n\pi_e - x)) f(x)dx \\
&\quad + \gamma\,(F\,(b(0)) - F(n\pi_e))
\end{aligned}$$

Suppose now that the optimal value is $\overline{m_\gamma} = 0$. This implies that $a(0) = n\pi_e$ and $b(0) = B$ is defined by :

$$\left\{ \begin{array}{c} V(0)\,[1 - \delta F(B)] = \int_{-\infty}^{B} [(n\pi_e - x)]\, f(x)dx + (\gamma - 1) \int_{n\pi}^{B} [(n\pi_e - x)]\, f(x)dx \\ \gamma B = \delta V(0) + n\pi_e + (\gamma - 1)\, n\pi_e \end{array} \right.$$

The optimal reserve will be $\overline{m_\gamma} > 0$, if the previous expression of $V_m'(0)$ in which we take $a(0) = n\pi_e$ is greater than $\frac{1}{\delta\rho}$. That is if:

$$F(n\pi_e) + \gamma(F(B) - F(n\pi_e)) > \frac{1}{\delta\rho}$$

In this case the optimal policy can be depicted as follows.

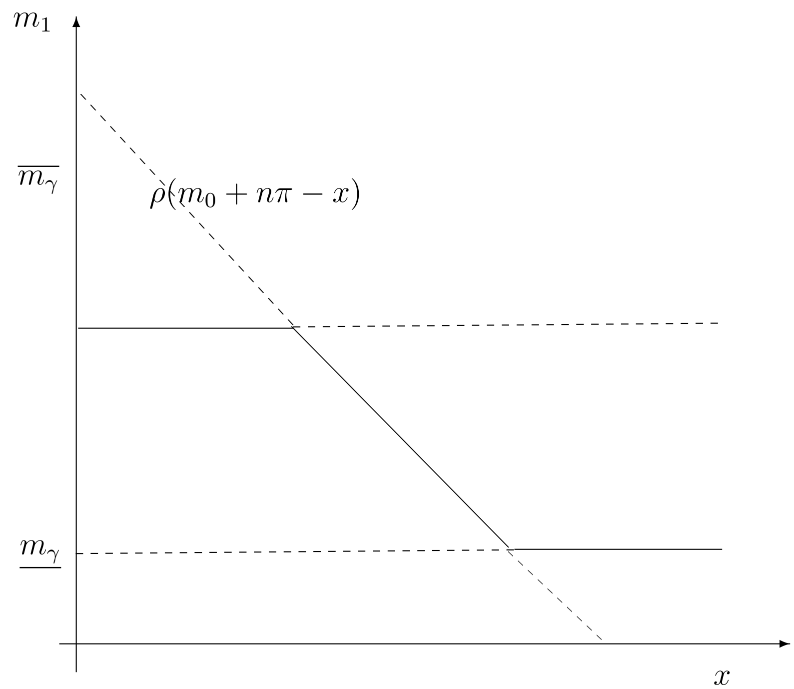


Fig. 1.— Optimal cash reserve as a function of claims

This barrier strategy can be related to the one defined in Décamps et al. (2008) on banking market and in Kulenko and Schmidli (2008) on insurance. It corresponds to (i) take dividend while cash reserve is above a bottom limit, (ii) neither take dividend nor issue new debt if the ex-post (cash) reserve is positive but below this limit and (iii) issue new shares in order to meet claims when the current reserve is insufficient. The main difference with Décamps et al. (2008) and Kulenko and Schmidli (2008) is however that, in their continuous framework, default never optimally occurs (they are therefore silent on commitment to recapitalize, central in our paper). Here, default endogeneously when the amount of cash needed to meet claims is higher than the present value of future income.

As the introduction of cost of issuance creates in incentive for cash reserve it seems interesting to analyze the effect of the cost (γ) on the probability of default. Noticing that the default threshold writes $b(m) = \frac{\delta}{\gamma}V(0) + m + n\pi_e$ and that $V(0)$ is decreasing with γ, we have the following proposition :

Proposition 5 *When the opportunity cost γ increases, both the precautionary reserve $\overline{m_\gamma}$ and the probability of default increase.*

Proof. see appendix ■

The two result of Proposition 5 may seem conflicting. However, the direct effect of an increase in γ, that is in the cost of issuance, is a decrease in recapitalization, that is an increase in the probability of default. Proposition 5 just states that the induces increase in precautionary reserve is not sufficient to counter this first-order effect. A simple consequence of this proposition is that the level of permanent capital is not here a signal of better solvency risk of the insurer. In the case of asymmetric information, the precautionary reserve is not aimed at diminishing the risk of insolvency but at diminishing the cost of recapitalization.

4.2.2. Information disclosure on cash reserve

The optimal Full Commitment policy cannot be non cooperatively implemented since there is no way for the shareholders to commit to recapitalize when losses are larger than the expected present value of future income. An interesting question would be to find a second best credible policy. In this second best world, if cash reserve are perfectly observed, it can be used as a commitment signal to some default probability.

Endogenously, shareholders provoke default when the present value of future income is lower than the claims, that is when x is larger than $m_0 + n\pi + \delta W_{SB}$, where W_{SB} is the optimal second best value of the firm.

We hence have:

$$
\begin{aligned}
V_{SB}(m_0) &= \int_0^{m_0+n\pi+\delta W_{SB}} (m_0 + n\pi + \delta W_{SB} - x)\, f(x)dx & (9)\\
&\quad \text{with } \pi \text{ the largest value of the premium } p \text{ such that,}\\
U(m_0,p) &= u(w-p)F(m_0+np+\delta W_{SB})\\
&\quad + \int_{m_0+np+\delta W_{SB}}^{+\infty} u\left(w + \frac{m_0 - x}{n}\right) f(x)dx \geq \underline{u} & (10)\\
\text{with} &: \delta W_{SB} = \max \delta V_{SB}(m) - \frac{1}{\rho} m & (11)
\end{aligned}
$$

The difference with the full commitment situation is that I is now constrained to be the interval $(0, m_0 + n\pi + \delta W_{SB}]$. This is due to the fact that the decision not to default must be ex ante credible.

Intuitively increasing m_0 allows to decrease the probability of default which in turn increases the value of the contract for the policyholders and hence allows an increase of $n\pi$. A natural question is then to compare the return of this "investment" with its cost.

Let $\pi_{\underline{u}}(m_0)$ the highest value of p solution of (10). We have:

Lemma 2 *$\pi_{\underline{u}}(m_0)$ is increasing, $\lim_{m_0\to+\infty} \pi_{\underline{u}}(m_0) = \underline{\pi}$, $\pi'_{\underline{u}}(m_0) \geq \frac{1-F(b)}{nF(b)}$, with $b = m_0 + n\pi_{\underline{u}}(m_0) + \delta W_{SB}$*

Now replacing π by $\pi_{\underline{u}}(m_0)$ in (9) and differentiating w.r.t m_0 gives :

$$
\frac{\partial V^{SB}}{\partial m_0}(m_0) = F(b)\left[1 + n\pi'_{\underline{u}}(m_0)\right].
$$

which is, because of lemma greater than 1 and is going to 1 when m_0 goes to infinity. Now, as at the optimum, the derivative has to equal $\frac{1}{\delta\rho}$, next proposition holds.

Proposition 6 *Let W_0, b_0 and π_0 be the solutions of*

$$
\left\{
\begin{array}{c}
W = \int_0^b (b-x)f(x)dx\\
b = n\pi + \delta W\\
u(w-\pi)F(b) + \int_b^{+\infty} u\left(w - \frac{x}{n}\right) f(x)dx = \underline{u}
\end{array}
\right. \qquad (12)
$$

Then, if

$$F(b_0) + (1 - F(b_0))\frac{E\left[u'\left(w - \frac{x}{n}\right)/x \geq b_0\right]}{u'(w - \frac{b_0}{n})} > \frac{1}{\delta\rho} \tag{13}$$

it is optimal to hold positive permanent cash reserve $m^{SB} > 0$

Thus, under condition (13), information disclosure on cash reserve, induces a permanent cash reserve, even when issuance is costless. In this case, the issuance and dividend policy consists in (i) paying dividend when reserve is above m^{SB} and (ii) recapitalize (that is to issue share) up to m^{SB} when reserve goes below m^{SB}. Still, if cash needed to set up reserve to m^{SB} is greater than the present value of future income, it is optimal for the insurer to default.

Noticing that W_0, b_0 and π_0 are the solution of the problem with asymmetric information (with $\gamma = 1$), it however turns out that under condition (13), information disclosure on cash reserve reduces the default probability (with respect to the case of asymmetric information). Because of the effect it as on the participation of policyholders (i.e. the higher acceptable premia), a (disclosed) positive cash reserve indeed enhances the value of firm, what in turn decrease the probability of default through our endogenous default probability.

However, as this default probability will still be strictly positive, disclosure of information appears to be insufficient to restore the first best. Even if it creates an incentive to always hold permanent cash reserve – contrarily to the full commitment case – information disclosure on cash reserve does not prevent for default to optimally occur. Let us therefore analyze if regulation can solve this commitment issue.

5. Regulation

In this section, we intuitively analyze the possible impact of two possible regulation schemes: capital requirement (that is the minimal amount of capital insurers have to hold to continue operating) and the setting up of a guarantee fund. As the lack of credibility on recapitalization has been show to be the main issue in reserve policy, the second option may be preferable.

5.1. The impact of minimal capital requirement

Let us first examine the possible implications of capital requirement on optimal capital reserve. Such a regulation rule (chosen in most countries) constraints the insurance companies to hold a minimal amount of capital $\underline{m}$. In our setting, this implies that recapitalization is needed as soon as claims exceed cash reserve *minus this ceiling* $\underline{m}$ (and therefore, more often than without regulation, for a given amount of initial capital). Moreover – under this scheme – when new shares (or debt) have to be issued, security holders need to build up capital reserve up to the required minimum (and no longer up to zero). As the amount of capital shareholders are willing to inject are still bounded by the present value of future returns these two mechanisms that (i) increases the need for recapitalization and (ii) potentially reduce the value of company, may lead to a perverse effect of capital requirement through an increase in the probability of default. Lastly, it appears that capital requirement may reduce the potential amount of "precautionary reserve" (by imposing early recapitalization) that could – as shown in Proposition 5 – reduce the cost of recapitalization.

The exact implication of reserve requirement (in particular on the value of the insurance company) however remains to be investigated and calls for further research. The analyze of the precise constrained program would for example allow us to discuss more precisely the impact of capital requirement on the value of the firm and to provide some interesting comparative statics on $\underline{m}$. Our analysis moreover seems to call for the study of an alternative policy that would consist in fixing a minimum amount of capital above which shareholders are prevented to take dividend but do not constrain them to recapitalize above zero (when claims exceed assets). Such a policy would create precautionary reserve without increasing the need for recapitalization.

5.2. A solution to the credibility issue: a guarantee to recapitalize

The questionable efficiency of capital requirement and the fact that the first best policy cannot be implemented because of a credibility issue lead us to consider an alternative form of regulation: the setting up of a guarantee fund that commits on recapitalization. We have shown in this paper that default occurs when security holders are reluctant to inject enough capital for the company to keep on operating. This issue will therefore be solved if a fund can commit – in these cases – to buy enough shares for the capital reserve not to be negative.

This however creates a typical moral hazard issue as it will then be easy for the shareholders to cheat on there capacity/willingness to add capital. They would then benefit from the capital injected by the fund without bearing the costs. This issue however disappears if we assume that this fund can infer the value of the company and can be eased by assuming that the guarantee fund conditions its intervention to a takeover of the company. Such a takeover would lead to a null value of the company (from the point of view of shareholders). Therefore the call for the fund would only be optimal (from the point of view of shareholders) only if the amount of needed cash exceeds the value of the company.

6. Conclusion

We highlight in the paper to role of the relationship between an insurer and its security holders in the need for capital and default regulation. We show that beside the informational issue between an insurer and its policyholders, regulation can be needed to solve an issue of credibility on recapitalization. It indeed appears in our work that the first best policy is not credible as it induces recapitalization in situation where shareholders have no incentive to inject capital.

Contrarily to existing literature we moreover show that an interior solution for capital reserve can be optimally chosen if recapitalization is costly. When the capital market is frictionless, that is when issuing new debt is costless, the optimal strategy consists in taking dividend as long as it is possible – because of agency cost – and to recapitalize each time it is needed (provided the future value of the company is larger than the invested capital). However, if issuing new debt is costly, it can be optimal to leave some capital in the company. The optimal strategy then consists in (a) taking dividend above a bottom limit, (b) neither take dividend nor issue new debt if the ex-post capital reserve is positive but below the limit and (c) issue new debt in order to meet claims when the current reserve is insufficient.

Taking into account the effect of default on policyholders, we show that the first best policy implies no default but no capital reserve. This policy however appears to be hardly implementable as it implies an ex-ante commitment to recapitalize which is not credible. An efficient regulation would therefore consist in making this commitment credible and therefore the setting up of a guarantee fund may be a more efficient regulation than capital requirement.

Is left for future research to analyze more precisely the optimal regulation. It would for example be interesting to evaluate optimal capital policy under capital requirement that is if capital reserve are constrained to be above a given level. We would then be able to define more exactly the efficient regulation. An other extension of interest would consist in studying the value of a share (and not of the company). Such a variant of our model may create an incentive for positive reserve (even with costless capital) as recapitalization – that is the issuance of new shares – would reduce the value of an existing share.

7. Appendix

Proof of Proposition 1.

We have :

$$\begin{aligned} V_{FC}(m_0) &= \max_{I,\pi} \int_I (m_0 + n\pi - x) f(x)dx + \delta W_{FC}\mu(I) \\ \text{s.t.} \quad & u(w-\pi)\mu(I) + \int_{I^C} u\left(w + \frac{m-x}{n}\right) f(x)dx \geq \underline{u} \end{aligned}$$

Let λ the Lagrange multiplier associated to the constraint and set $I = [0, b)$,.

The two partial derivatives of the lagrangian L are:

$\frac{1}{F(b)} \frac{\partial L}{\partial \pi} = n - \lambda u'(w - \pi)$
$\frac{1}{f(b)} \frac{\partial L}{\partial \pi} = (m_0 + n\pi + \delta W_{FC} - b) + \lambda \left[u(w-\pi) - u\left(w + \frac{m-b}{n}\right)\right]$

Suppose that the optimal premium is π^*. We have hence $u'(w - \pi^*) = \frac{n}{\lambda}$. The second derivative is :

$$(m_0 + n\pi^* + \delta W_{FC} - b) + n\frac{\left[u(w-\pi^*) - u\left(w + \frac{m-b}{n}\right)\right]}{u'(w-\pi^*)}$$

that is a convex function of b with a minimum at the point where $u'\left(w + \frac{m-b}{n}\right) = u'(w - \pi^*)$ that is on $b = m_0 + n\pi^*$. The value of this derivative at this point is hence exactly δW_{FC}, which is essentially positive. That means that the second derivative is always positive and hence that $b = +\infty$. ■

Proof of Lemma 1.

We have, as $\overline{m_\gamma}$ and $\underline{m_\gamma}$ respectively maximize $\delta V_{NC}(m) - \frac{1}{\rho}m$ and $\delta V_{NC}(m) - \frac{\gamma}{\rho}m$:

$$\begin{cases} \delta V_{NC}(\overline{m_\gamma}) - \frac{1}{\rho}\overline{m_\gamma} \geq \delta V_{NC}(\underline{m_\gamma}) - \frac{1}{\rho}\underline{m_\gamma} \\ \delta V_{NC}(\underline{m_\gamma}) - \frac{\gamma}{\rho}\underline{m_\gamma} \geq \delta V_{NC}(\overline{m_\gamma}) - \frac{\gamma}{\rho}\overline{m_\gamma} \end{cases}$$

Adding up these two inequalities gives :$(\gamma - 1)\left(\overline{m_\gamma} - \underline{m_\gamma}\right) \geq 0$.

Moreover $\delta W(g) = \max_{m\geq 0} \delta V(m) - \frac{g}{\rho}m$ is the supremum of decreasing affine (and hence convex) functions. It is hence a convex decreasing function, such that $\delta W'(g) = \frac{-m_g}{\rho}$ (almost everywhere). ■

Proof of proposition 4.

Assume that $\delta V_{NC}(m) - \frac{1}{\rho}m$ and $\delta V_{NC}(m) - \frac{\gamma}{\rho}m$ are functions increasing before their maximum and decreasing after.

It is easy to see that $S(m) = \delta V_{NC}(m) + \varphi\left(m_0 + n\pi_e - x - \frac{1}{\rho}m\right)$ is maximized for $m = M(x, m_0) = \max\left(\underline{m_\gamma}, \min(\rho(m_0 + n\pi_e - x), \overline{m_\gamma})\right)$.

Indeed when $m < \rho(m_0 + n\pi_e - x)$, $S(m) = \delta V_{NC}(m) - \frac{1}{\rho}m + m_0 + n\pi_e - x$, which is hence maximized for $m = \min(\rho(m_0 + n\pi_e - x), \overline{m_\gamma})$. When $m > \rho(m_0 + n\pi_e - x), S(m) = \delta V_{NC}(m) - \frac{\gamma}{\rho}m + \gamma\left(m_0 + n\pi_e - x\right)$, which is maximized for $m = \max\left(\underline{m_\gamma}, \rho(m_0 + n\pi_e - x)\right)$.

Moreover, recall that :

$$V_{NC}(m_0) = \max_{I, m_1(.)\geq 0} \int_I \varphi\left(m_0 + n\pi_e - x - \frac{1}{\rho}m_1(x)\right) f(x)dx + \int_I \left(\delta V_{NC}(m_1(x))\right) f(x)dx$$

By the envelop theorem :

$$V'_{NC}(m_0) = \int_I \varphi'\left(m_0 + n\pi_e - x - \frac{1}{\rho}m_1(x)\right) f(x)dx$$

Where $\varphi'\left(m_0 + n\pi_e - x - \frac{1}{\rho}m_1(x)\right) = 1$ for $x < m_0 + n\pi_e - \frac{1}{\rho}\overline{m_\gamma}$, $\varphi'(.) = \gamma$ for $x > m_0 + n\pi_e - \frac{1}{\rho}\underline{m_\gamma}$, and some values between 1 and γ for x in between. Hence $V'_{NC}(m_0)$ has an expectation of values between 1 and γ. Therefore $V'_{NC}(m_0) \leq \gamma$. This implies that $\delta V'_{NC}(m_0) - \frac{\gamma}{\rho}$ is strictly negative. That means that $\underline{m_\gamma} = 0$. ■

Proof of proposition 5.

We know that:

$$\begin{aligned} V = & \max_{a \leq m + n\pi \leq b} \int_{-\infty}^{a} \left[\delta W^* + (m_0 + n\pi_e - x)\right] f(x)dx \\ & + \int_{a}^{m_0 + n\pi_e} \delta V(\rho(m_0 + n\pi_e - x)) f(x)dx \\ & + \int_{m_0 + n\pi_e}^{b} \left[\delta V(0) + \gamma(m_0 + n\pi_e - x)\right] f(x)dx \end{aligned}$$

With the envelop theorem the derivative of V with respect to γ is hence:

$$V'_\gamma = \int_A^{b^*(m)} \left[\delta V'_\gamma(0) + (m_0 + n\pi_e - x)\right] f(x)dx.$$

In particular:

$$V'_\gamma(0) = \int_{m_0+n\pi_e}^{b^*(0)} \left[\delta V'_\gamma(0) + (n\pi_e - x)\right] f(x)dx,$$

which give :

$$V'_\gamma(0)\left(1 - \delta\left(F(b^*(0)) - F(m_0 + n\pi_e)\right)\right) = \int_{m_0+n\pi_e}^{b^*(0)} (n\pi_e - x)\, f(x)dx,$$

and implies $V'_\gamma(0) \leq 0$. ■

References

Avanzi, B., "Strategies for Dividend Distribution: A Review" (January 20, 2009). *North American Actuarial Journal*, 13(2), pp. 217-251, 2009.

Blazenko, G., Pavlov, A. and Parker, G., "Financial Risk Theory for a Regulated Insurer", *working paper*, 2007.

Blazenko, G., Pavlov, A. and Parker, G., "Insurer Risk Management and Shareholder Wealth", *working paper*, 2008.

Borch, K., "Is Regulation of Insurance Companies Necessary" In Geld, Banken, Versicherungen, H. Gŏppland, and R. Henn (Eds.), Kőnigstein, 1981.

De Finetti, B., "Su un'Impostazione Alternativa della Teoria Collettiva del Rischio". *Transactions of the XVth International Congress of Actuaries*, pp. 43343, 1957.

Décamps, J.-P., Mariotti, T., Rochet, J.-C. and Villeneuve, S., "Free Cash-Flow, Issuance Costs and Stock Volatility", *IDEI working paper*, 2008.

Finsinger, J. and Pauly, M., "Reserve Levels and Reserve Requirements for Profit Maximising Insurance Firms" In Risk and Capital, G. Bamberg and K. Spremann (Eds.), Berlin: Springer, 1984.

Gerber, H., "A Characteristic Property of the Poisson Distribution", *American Statistician*, 33(2), pp. 8586, 1979.

Gerber, H., and Shiu, E., "Optimal Dividends: Analysis with Brownian Motion", *North American Actuarial Journal*, 8(1), pp. 120, 2004.

Gomes, J., "Financing Investment", *The American Economic Review*, 91(5), 2001.

Kulenko, N. and Schmidli, H., "Optimal dividend strategies in a Cramér–Lundberg model with capital injections", *Insurance: Mathematics and Economics*, 43, pp. 270–278, 2008.

La Porta, R., Lopez-De-Silanes, F., Shleifer, A. and Vishny, R., "Agency Problems and Dividend Policies around the World", *The Journal of Finance*, 55 (1), 2000.

Leland, H. and Toft, K., "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads", *The Journal of Finance*, Vol. 51(3), pp. 987–1019, 1996

Munch, P. and Smallwood, D., "Theory of Solvency Regulation in the Property and Casualty Insurance Industry" In G. Fromm, ed. Studies in Public Regulation. Cambridge, Mass., MIT Press, 1982.

Rees, R., Gravelle, H. and Wambach, A., "Regulation of insurance markets", *The Geneva Papers on Risk and Insurance Theory*, 24, pp. 55–68, 1999.

Rochet, J.-C., and Villeneuve, S., "Corporate Portfolio Management", *Annals of Finance*, vol. 1(3), pp. 225-243, 2005.